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TRANSIENT FIELDS PRODUCED IN HETEROGENEOUS BODIES BY ELECTROMAG--ETC(U)
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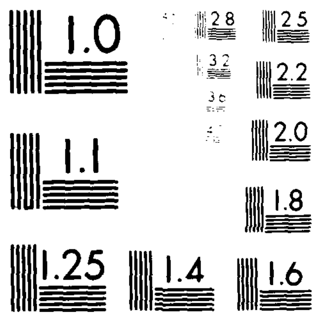

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Report SAM-TR-81-12

TRANSIENT FIELDS PRODUCED IN HETEROGENEOUS BODIES BY ELECTROMAGNETIC PULSES

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October 1981

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Brooks Air Force Base, Texas 78235

NOTICES

This final report was submitted by the Department of Mathematics, University of Nebraska, Lincoln, Nebraska, under contract F33615-78-D-0629, Task 0022, job order 7757-01-83, with the USAF School of Aerospace Medicine, Aerospace Medical Division, AFSC, Brooks Air Force Base, Texas. Earl L. Bell (USAFSAM/BRM) was the Laboratory Project Scientist-in-Charge.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An exact analytical technique for determining induced electromagnetic fields in a one-dimensional medium is developed. The incident field, which impinges normally on the medium, is an electromagnetic pulse of arbitrary shape and duration. The medium is of finite depth and can have arbitrarily specified and nonconstant permittivity and conductivity. This analytical result is implemented in a computer program called HATS (High Accuracy Temporal Solution), which is virtually free of numerical dispersion.

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20. ABSTRACT (continued)

Several sample problems are formulated and then solved via this program. Appendix B supplies a manual that details use of the program and includes a source listing of HATS, sample input, and plots.

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TRANSIENT FIELDS PRODUCED IN HETEROGENEOUS BODIES BY ELECTROMAGNETIC PULSES

STATEMENT OF THE PROBLEM

This effort examines the question of electromagnetic field structure induced in heterogeneous bodies by pulse electromagnetic radiation. Of special interest is the case of a sinusoidally varying incident field of finite duration.

To determine the structure of induced electromagnetic fields, the problem was modeled mathematically. The assumptions made and the resulting model are given below. As the model is described, the nature of the media and incident fields to be considered will be made clear.

The medium considered in this work is one dimensional and of finite depth L. Thus, it is assumed to vary spatially only in the z direction, extending from z=0 to z=L. The permittivity, $\varepsilon(z)$, and conductivity, $\sigma(z)$, of the medium are piecewise continuous functions as pictured, for example, in Figure 1. By allowing ε and σ to be piecewise continuous, a layered medium can be considered. Free space is assumed to exist outside of the medium, so $\varepsilon(z)=\varepsilon_0$ and $\sigma(z)=0$ for $z<0$ and $z>L$. Finally, the medium is assumed to be non-dispersive in that the permittivity and conductivity are independent of the induced field.

An electromagnetic wave propagating along the z-axis normal to the medium has transverse electric field, $E(z,t)$, satisfying

$$E_{zz}-\varepsilon(z)\mu_0 E_{tt}-\sigma(z)\mu_0 E_t=0, \quad -\infty<z<\infty, \quad -\infty<t<\infty \tag{1}$$

which follows from Maxwell's equations. Here, t denotes time, μ_0 is the constant magnetic permeability, and subscripts denote partial derivatives. The

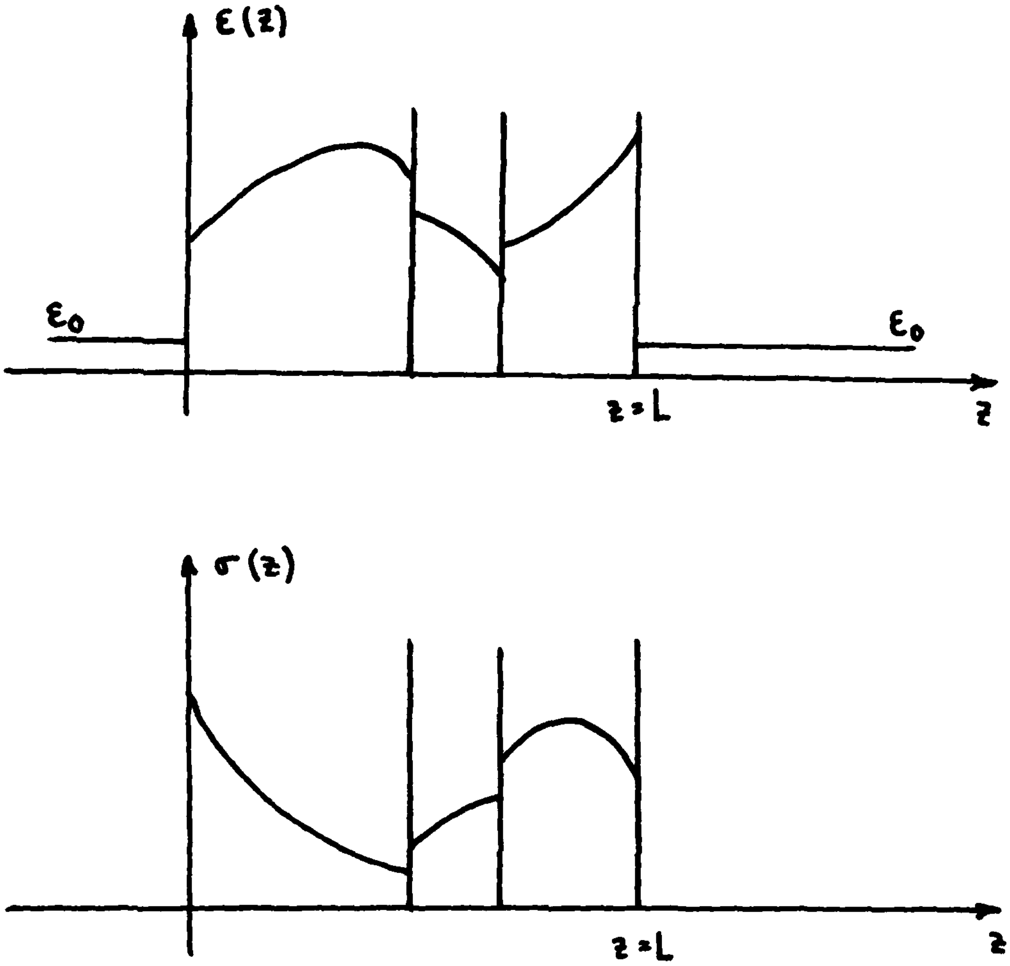

Figure 1. Example of permittivity and conductivity profiles for a one-dimensional, three-layer medium.

incident electromagnetic pulse, E^i, is assumed to propagate from left to right and first impinges on the medium at time t=0. Thus, the initial condition for equation (1) is

$$E(z,t)=E^i(z-ct) \text{ for } t<0 \qquad (2)$$

where $E^i(z-ct)=0$ for $z-ct\geq 0$ and c is the speed of light in free space. Figures 2 and 3 show what some typical incident pulses may look like.

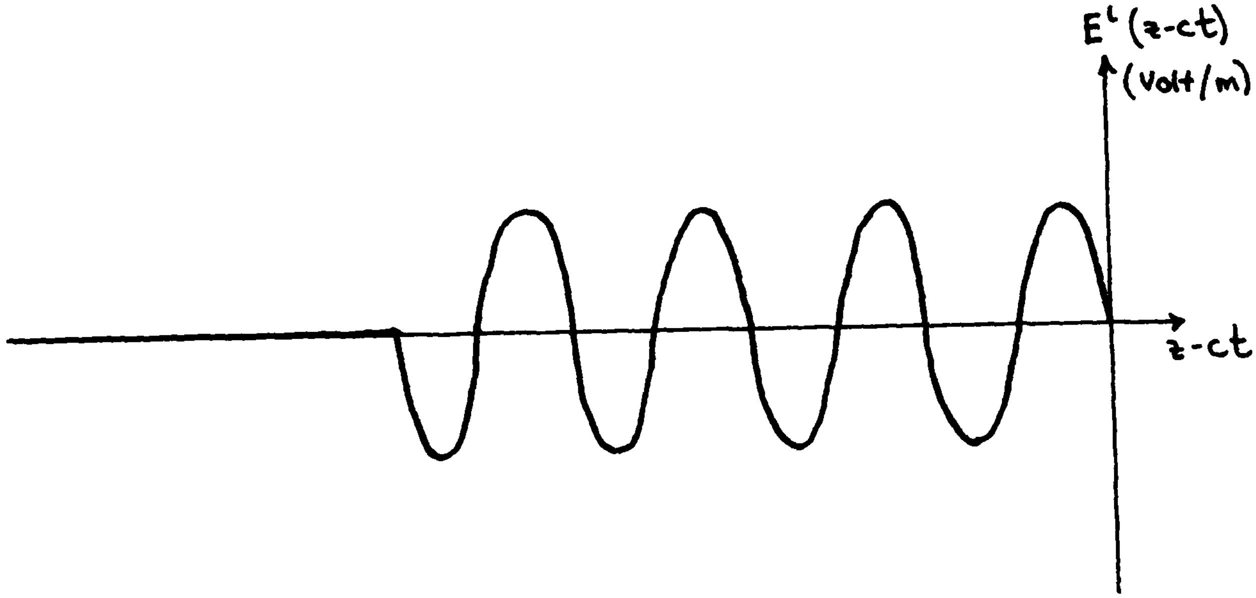

Figure 2. A sinusoidal incident pulse of finite duration.

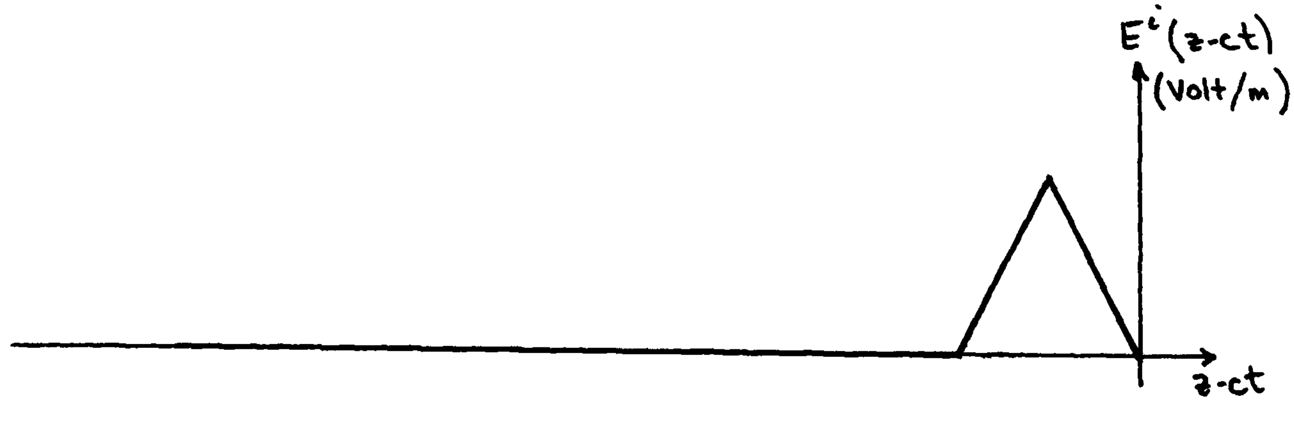

Figure 3. A spike incident field.

The problem considered, then, is that of determining the solution $E(z,t)$ of the system of equations (1) and (2) for $0 \leq z \leq L$, $t > 0$ for any given $E^i(z-ct)$, $\varepsilon(z)$, $\sigma(z)$.

SOLUTION TECHNIQUE

The problem modeled above has been solved numerically by the HATS computer program (High Accuracy Temporal Solution). A detailed description of the mathematical foundations of the numerical methods used in the program can be found in reference 8, and preliminary results are given in references 4-7. A brief summary of reference 8 is given in Appendix A. A description of the use and capabilities of this program is given in Appendix B, which is a self-contained user's manual for HATS.

Note that the numerical solution is based on exact analytical representations of the fields involved, with no recourse to low frequency approximations. The technique used has been found to be highly stable.

SAMPLE PROBLEM

As an example of the results that can be obtained via the HATS program, consider the three-layer medium shown in Figure 4, where it is first assumed that $\sigma(z) \equiv 0$. A 1000-MHz incident field of 20-ns duration impinges on the medium from the left at time $t=0$. Thus, at some time $t<0$ the incident field looks like that shown in Figure 5.

The resulting field inside the medium was generated by the HATS program for the time interval $t=0$ to $t=25$ ns. Figures 6, 7, and 8 show the internal fields as a function of time at three specific locations ($z=.714$ cm, $z=1.067$ cm, $z=2.100$ cm) in the medium. The locations $z=.714$ cm and $z=2.100$ cm (Figs. 6 and 8) were chosen because they represent in some sense the "worst case" behavior, in that the transients at these points had the greatest amplitude relative to steady state amplitude of any points in the medium. Figure 7 ($z=1.067$ cm) represents the "best case" behavior in that the ratio of transient amplitude to steady amplitude was the smallest. At all points in the medium, transient amplitude was greater than steady state amplitude.

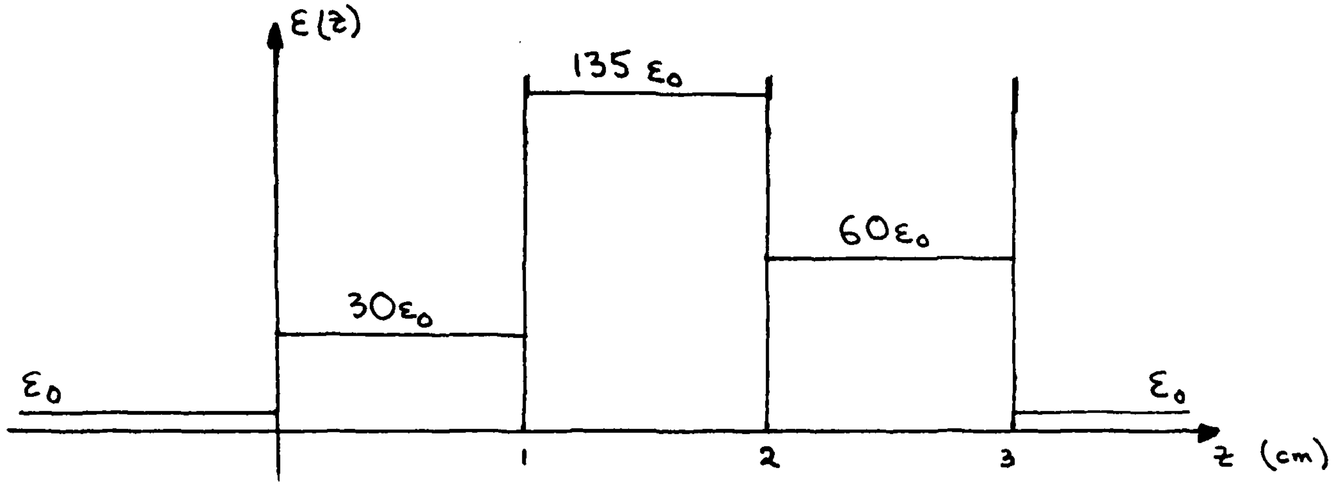

Figure 4. Permittivity profile for the sample problem.

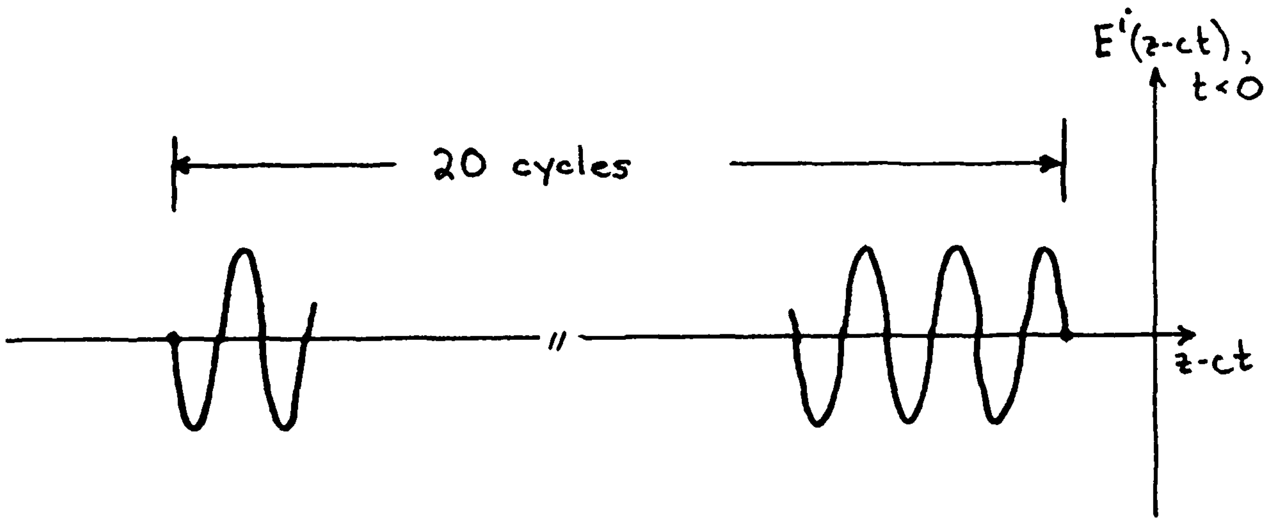

Figure 5. Incident field for the sample problem. (20-ns pulse, 1000 MHz)

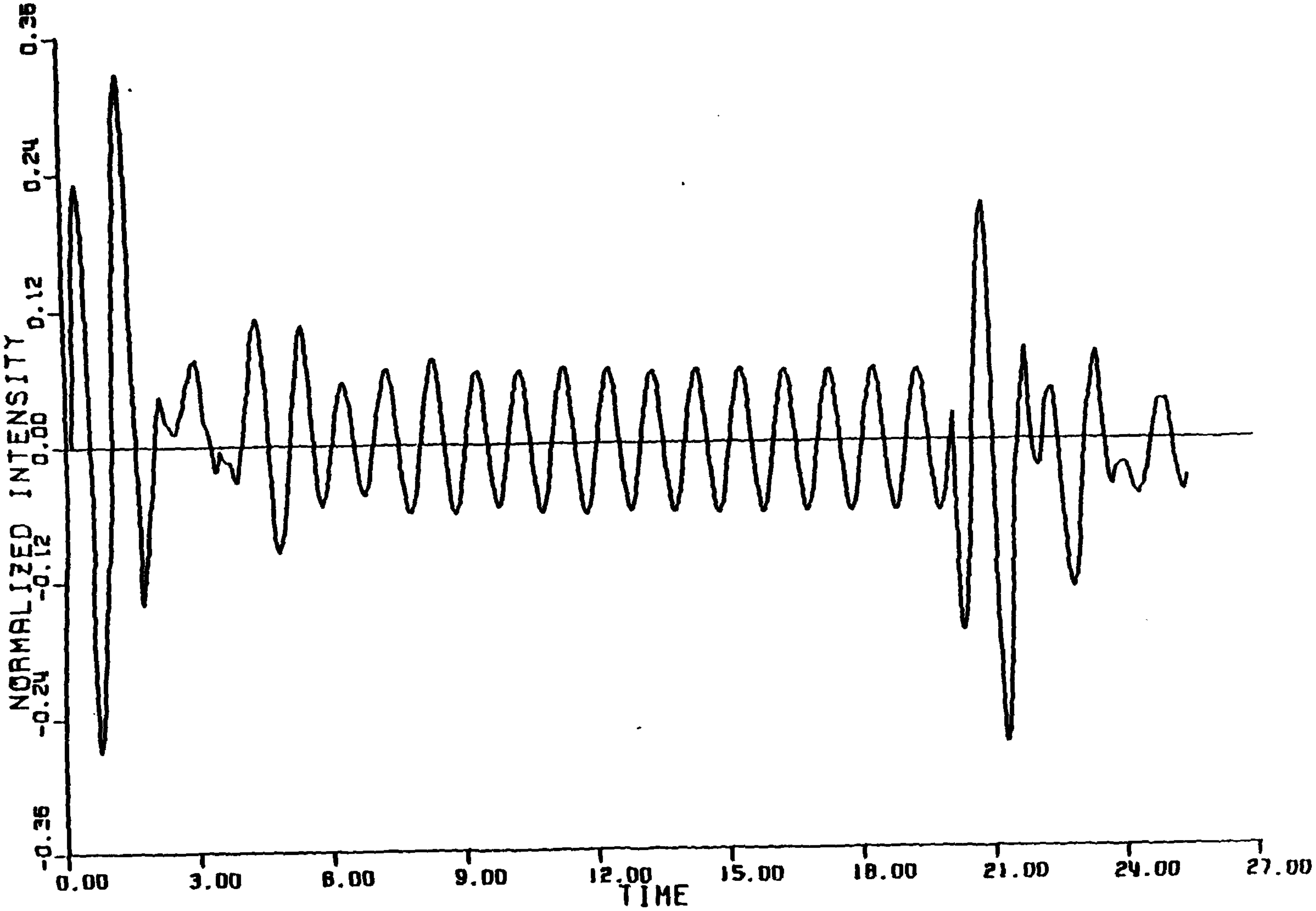

Figure 6. Internal field (z=0.714 cm) vs. time (ns), zero conductivity.

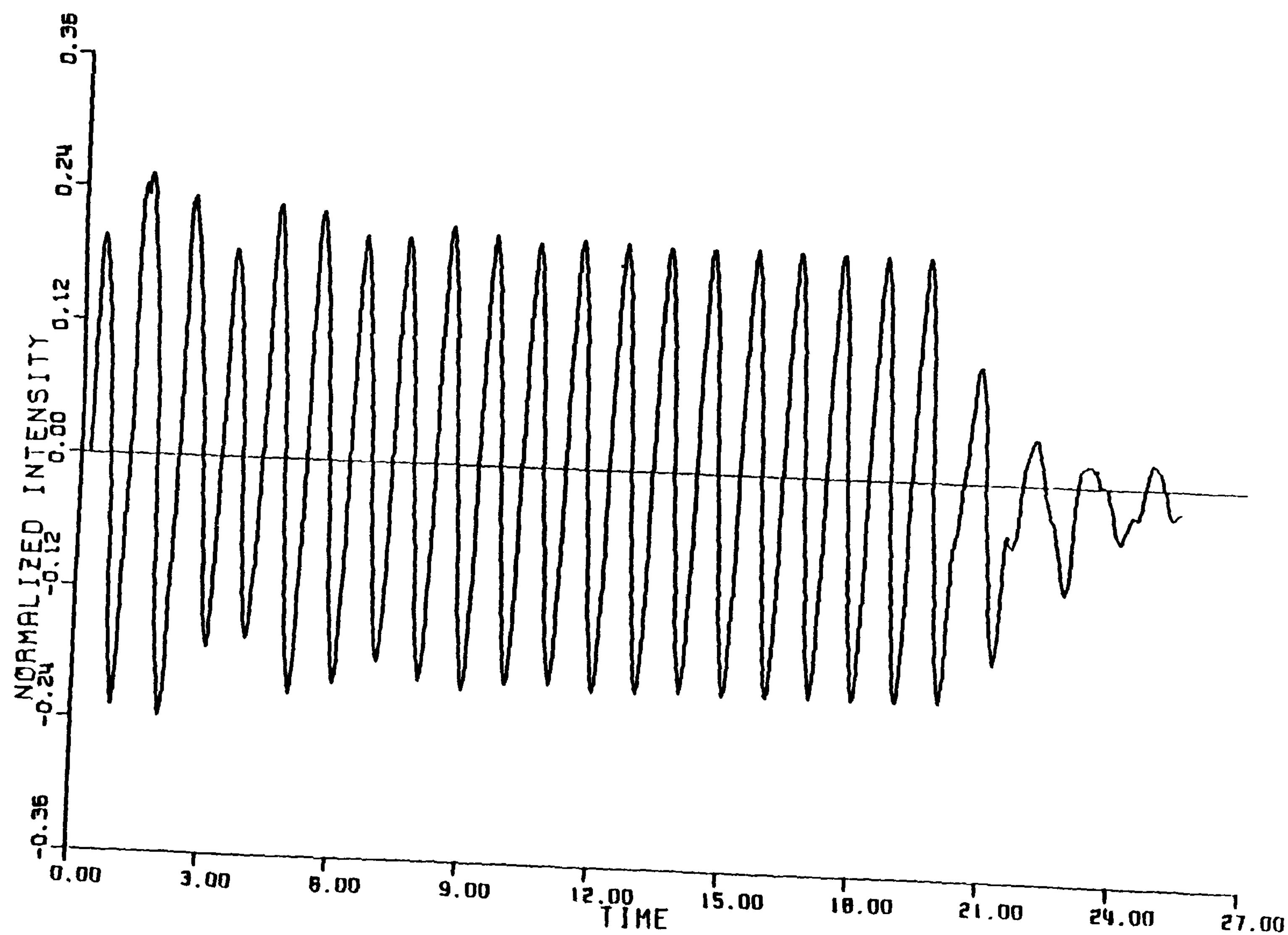

Figure 7. Internal field (z=1.067 cm) vs. time (ns), zero conductivity.

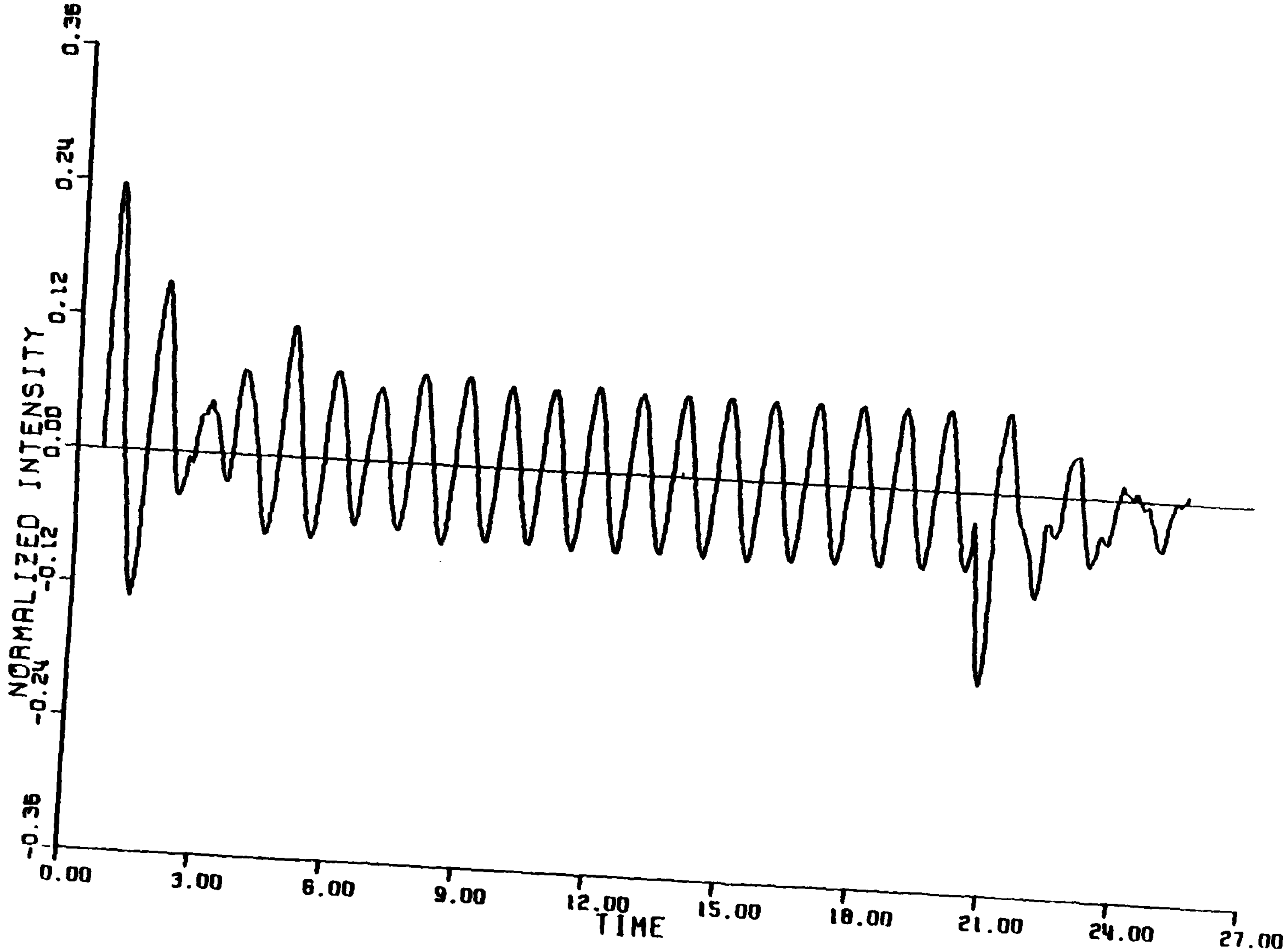

Figure 8. Internal field (z=2.100 cm) vs. time (ns), zero conductivity.

Now consider a medium with permittivity profile given in Figure 5 and conductivity profile given in Figure 9. The fields at z=.714 cm, z=1.067 cm, z=2.100 cm (Figs. 10-12) fairly accurately give worst and best behavior. Again, transient amplitude was everywhere greater than steady amplitude, although the effect of nonzero conductivity reduces the ratio of transient to steady amplitude.

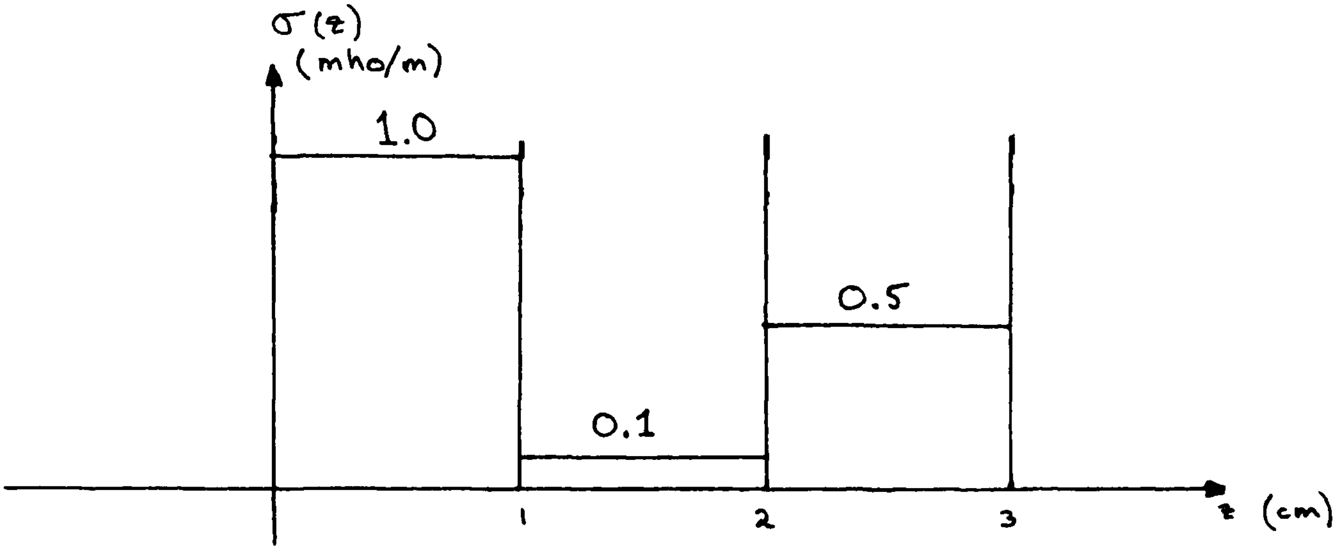

Figure 9. Conductivity profile for the sample problem.

Additional information regarding these plots (how the specific values of z were chosen, views of the entire field in the medium at particular times, etc.) and a detailed discussion of how they were obtained are given in Appendix B. The problem studied in this section is only a sample problem; any permittivity and conductivity profiles and incident field can be input into the HATS program, modulo the restrictions called out in Appendix B.

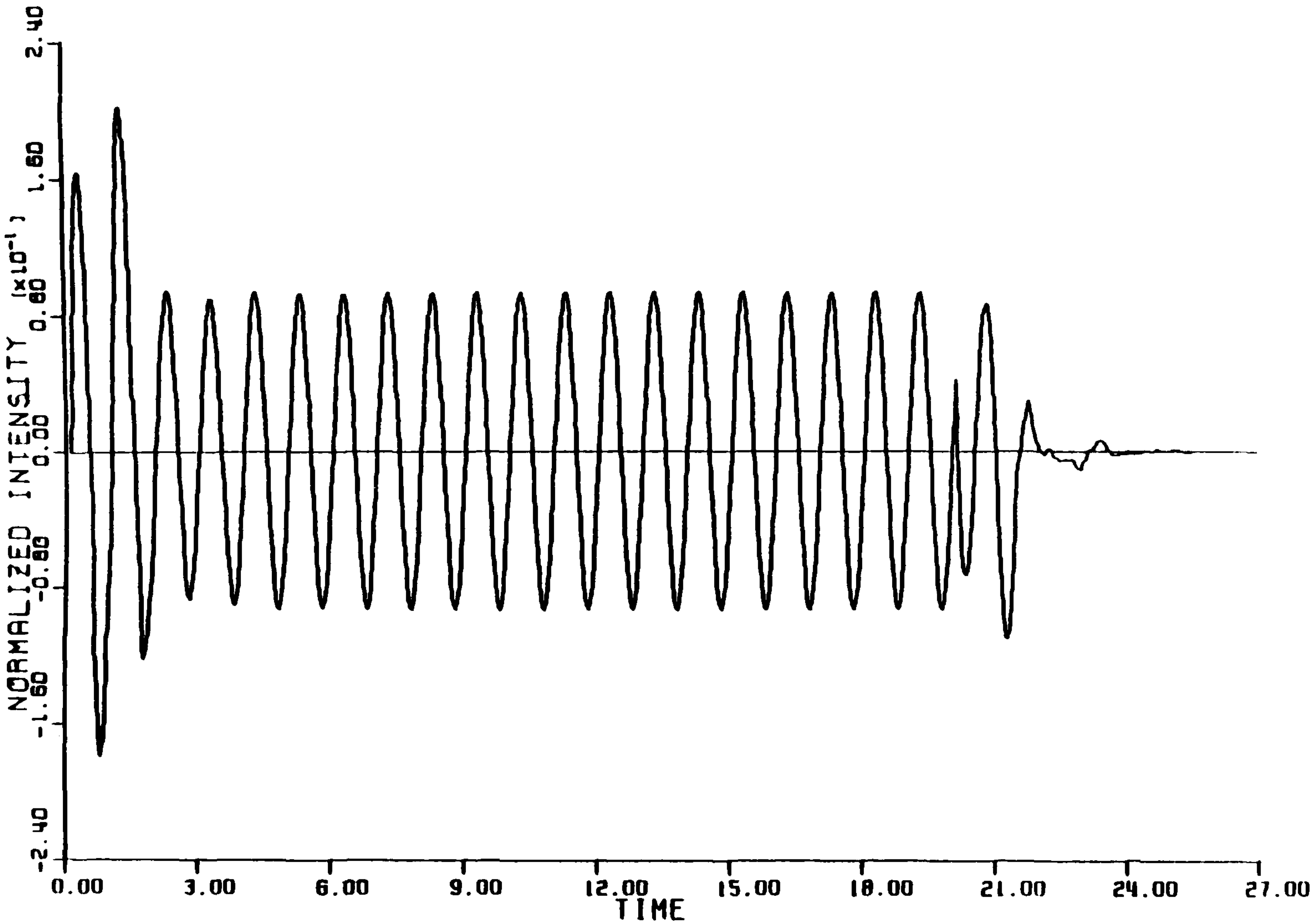

Figure 10. Internal field (z=0.714 cm) vs. time (ns), nonzero conductivity.

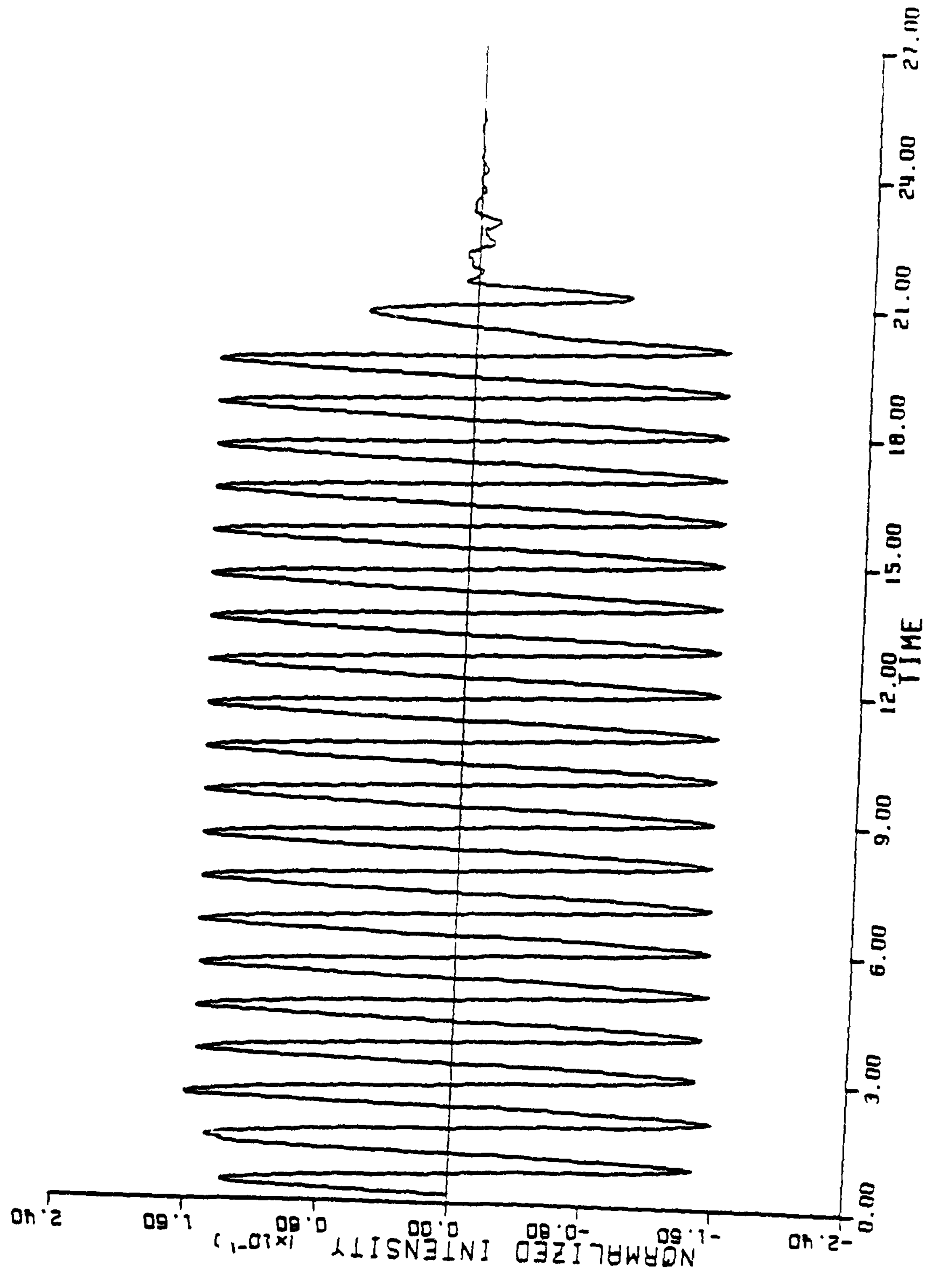

Figure 11. Internal field (z=1.067 cm) vs. time (ns), nonzero conductivity.

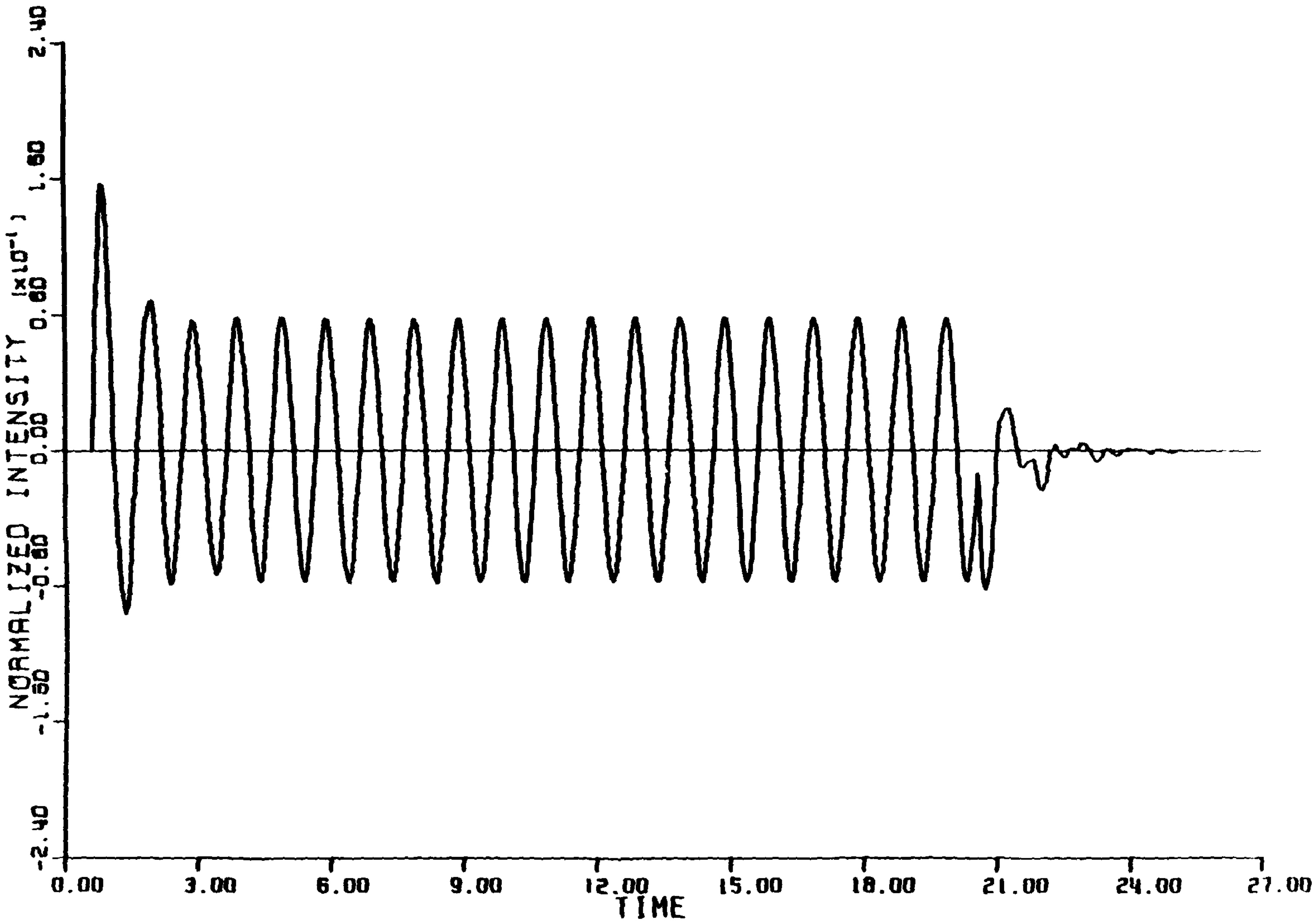

Figure 12. Internal field (z=2.100 cm) vs. time (ns), nonzero conductivity.

CONCLUSIONS

Use of the HATS computer program has shown that relatively large amplitude transients can occur in a medium when a transient signal, such as that from a phased array system, impinges on the medium. Although the model used to determine this is a fairly simple one, the pattern shown here should extend to more realistic situations. Nonetheless, further work is needed to definitely establish the presence and magnitude of these transients for other geometries and for the important case of a frequency dependent medium, such as human tissue.

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APPENDIX A. OUTLINE OF SOLUTION TECHNIQUE

The problem considered in this work is modeled by

$$E_{zz}-\varepsilon(z)\mu_0 E_{tt}-\sigma(z)\mu_0 E_t=0,\ -\infty<z<\infty,\ -\infty<t<\infty. \tag{A.1}$$

It is assumed that $\varepsilon(z)$ and $\sigma(z)$ are given and that $\varepsilon(z)=\varepsilon_0$, $\sigma(z)=0$ for $z<0$ and $z>L$. Finally, the solution of equation (A.1) is given by

$$E(z,t)=E^i(z-ct)\ \text{for}\ t<0 \tag{A.2}$$

where $E^i(z-ct)=0$ for $z-ct\geq 0$. The solution of the initial value problem (A.1, A.2) must be found for $0\leq z\leq L$ and $t>0$.

To facilitate the numerical solution of equation (A.1), the transformations

$$x=x(z)=\left(\int_0^z\{\varepsilon(s)\mu_0\}^{\frac{1}{2}}ds\right)/\ell \tag{A.3}$$

$$\tau=t/\ell \tag{A.4}$$

$$u(x,\tau)=E(z,t) \tag{A.5}$$

are used to convert equation (A.1) to

$$u_{xx}-u_{\tau\tau}+A(x)u_x+B(x)u_\tau=0,\ -\infty<x<\infty,\ -\infty<\tau<\infty \tag{A.6}$$

where

$$A(x)=-\ell\frac{d}{dZ}\{\varepsilon(z)\mu_0\}^{-\frac{1}{2}}$$

$$B(x)=-\ell\sigma(z)/\varepsilon(z)$$

and ℓ is such that $x(L)=1$. In terms of this transformed problem, the solution $u(x,\tau)$ of equation (A.6) must be found for $0\leq x\leq 1$, $\tau>0$ given that

$$u(x,\tau)=u^i(x-\tau) \text{ for } \tau<0 \tag{A.7}$$

where $u^i(\tau)=E^i(c\ell\tau)$.

The advantage of the change of variables (A.3)-(A.5) is that the characteristics of equation (A.6) are straight lines. On the other hand, while the solution E of equation (A.1) has continuous first derivatives, the solution u of equation (A.6) has discontinuities in its x derivative. To see this, assume that $\varepsilon(z)$ is discontinuous at the points $z=z_i$, $i=0, 1, \ldots, NL$ where $0=z_0<z_1<z_2<\ldots<z_{NL}=L$. Let $x_i=x(z_i)$ for $i=0, 1, \ldots, NL$. Then the change of variable (A.3) implies that

$$c_i u_x(x_i+, \tau)=u_x(x_i-, \tau) \tag{A.8}$$

for $i=0, 1, \ldots, NL$ where

$$c_i=\{\varepsilon(z_i+)/\varepsilon(z_i-)\}^{\frac{1}{2}}.$$

One technique for solving the problem (A.6)-(A.8) is by using finite differences [1]. However, in using an explicit differencing scheme on this problem, roundoff error accumulated quickly and prevented accurate solutions for incident pulses of realistic duration. An alternate approach is to use the time-domain integral equation technique of Bolomey et al. [3] with suitable modifications to take into account the finite depth of the medium. However,

this technique proved to be susceptible to numerical dispersion in that truncation error propagates through the solution at nonphysical speeds, causing spreading of the fields. Again, for pulses of realistic duration, this creates highly unreliable solutions.

The above two solution techniques are based on the familiar space-time diagram shown in Figure A-1, which displays the fact that the solution u at the point (x,τ) depends on the solution in a certain interval (the domain of dependence) on the x-axis determined by the characteristic lines emanating from (x,τ). Note, however, that an equally valid solution technique is afforded through the space-time diagram of Figure A-2. To understand the data required for the formulation of the problem suggested by Figure A-2, note that the incident electromagnetic field $E^i(z-ct)$ gives rise to a transmitted field $E^t(z-ct)$ for $z>L$. In the transformed problem (A.6)-(A.8), the incident plane wave $u^i(\tau)=E^i(c\ell\tau)$ produces a transmitted plane wave $u^t(x-\tau)$ where $u^t(\tau)=E^t(L-c\ell+c\ell\tau)$ and $u^i(\tau)=u^t(\tau)=0$ for $\tau>0$. Since $u(1,\tau)-u^t(1-\tau)$, it follows that the data required for the formulation of the problem shown in Figure A-2 is $u^t(\tau)$ for $\tau<0$. The advantage of this formulation over that suggested by Figure A-1 is that exact analytical expressions exist for the solution $u(x,\tau)$ in terms of the data u^t and these expressions give rise to highly accurate numerical schemes.

Pursuing the approach suggested by Figure A-2, it is first necessary to determine the transmitted field u^t corresponding to a given incident field u^i. In reference 5 these fields are shown to be related by the equation

$$u^i(\tau)=\sum_{j=1}^{2^{NL}} w_j u^t(\tau+r_j)+\int_\tau^0 W(\tau-s)u^t(s)ds, \quad \tau<0. \tag{A.9}$$

The constants w_j and r_j are known and are determined by the properties of the medium (formulas are given in reference 5). The kernel function, W, also depends only on the properties of the medium and is independent of the fields u^i, u^t.

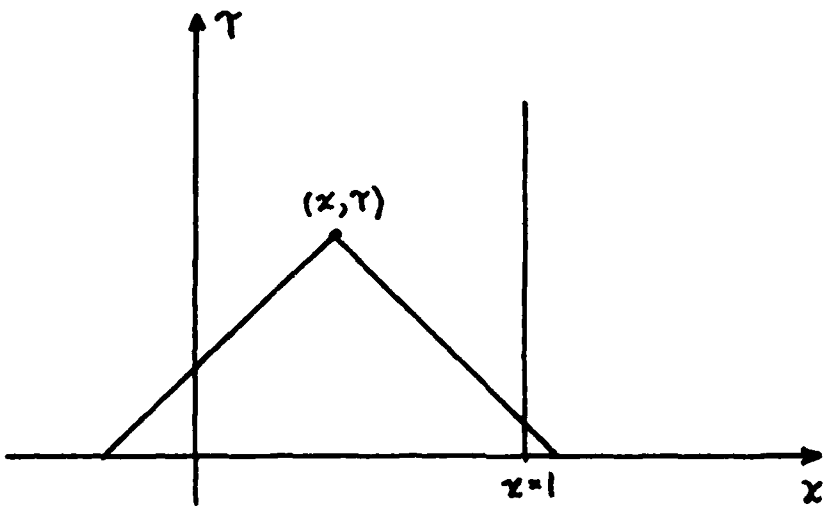

Figure A.1. Space-time diagram showing domain of dependence along line $\tau=0$.

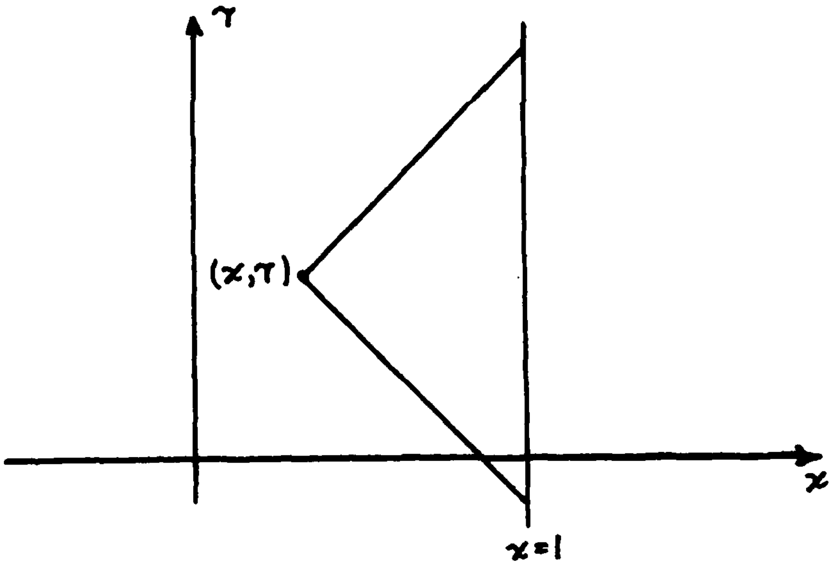

Figure A.2. Space-time diagram showing domain of dependence along line $x=1$.

Assuming for the moment that W is known, equation (A.9) is a delay Volterra integral equation for u^t. Thus, it is equivalent to a Volterra integral equation of the second kind (as shown in reference 6), so stable numerical techniques for solving equation (A.9) for u^t are available (see reference 2). Consequently, the data required for the formulation in Figure A-2 can be considered known.

Reference 8 shows that for any x in the interval $[x_{NL-1}, 1]$ and for any τ, the solution $u(x,\tau)$ of equation (A.6) is given by

$$2u(x,\tau)=\exp\{1/2\int_x^1(A(s)-B(s))ds\}\{(c_{NL}+1)u^t(x-\tau)$$

$$-(c_{NL}-1)u^t(2-x-\tau)\exp\{\int_x^1 B(s)ds\}$$

$$+\int_x^{2-x}u^t(y-\tau)N(x,y,1,)dy\}. \tag{A.10}$$

Here, $N(x,y,1)$ is the solution of the characteristic initial value problem

$$N_{xx}-N_{yy}+B(x)(N_x+N_y)+D_+(x)N=0,\ x_{NL-1}\leq x\leq x_{NL}=1, \tag{A.11}$$

$$2N(x,x,1)=c_{NL}B(1)-A(1)-(c_{NL}+1)\int_x^1 D_+(s)ds, \tag{A.12}$$

$$2N(x,2-x,1)=(c_{NL}B(1)-A(1)+(c_{NL}-1)\int_x^1 D_-(s)ds)\ \exp\{\int_x^1 B(s)ds\} \tag{A.13}$$

where

$$A(1)=A(1-),\ B(1)=B(1-)$$

and

$$D_\pm=1/4(B^2-A^2)+1/2(-A'\pm B').$$

The problem (A.11)-(A.13) can be easily solved numerically by the method of characteristics [1]. Thus, $u(x,\tau)$ can be determined from equation (A.10) for any x in $[x_{NL-1}, 1]$ by means of a simple quadrature. To determine $u(x,\tau)$ for x in $[x_{NL-2}, x_{NL-1}]$, reference 8 shows that a system of formulas analogous to equations (A.10)-(A.13) utilizes the data $u(x_{NL-1}, \tau)$. Continuing in this manner, quadrature formulas can be used to determine $u(x,\tau)$ for any x in the interval [0,1]. In the program HATS, the trapezoidal rule is used to perform these quadratures.

The remaining point to be considered is determination of the kernel function, W, which appears in equation (A.9). If the transmitted field is a Dirac delta function,

$$u^t(\tau)=\delta(\tau) \tag{A.14}$$

then, according to equation (A.9), the "incident field" that would generate such a transmitted field is the kernel W, modulo delta function singularities. But if the transmitted field is assumed to be given by equation (A.14), then the technique outlined above enables one to determine the corresponding field $u(0,\tau)$ for all τ. From this it is possible to separate out the incident field that generates (A.14) and hence obtain W. Again, details are given in reference 8.

APPENDIX B. USE OF THE HATS PROGRAM

I. INTRODUCTION

The following general description is meant to be a user's guide to the program HATS (High Accuracy Temporal Solution) and does not dwell on mathematical details other than what is needed to properly use the program.

Briefly stated, this program allows a user to cause an incident plane electromagnetic wave to impinge normally on a medium of finite depth and to determine the resulting scattered and internal fields. The medium is assumed to be one-dimensional (i.e., stratified in the z direction) and "linear" in that its permittivity, $\varepsilon(z)$, and conductivity, $\sigma(z)$, are independent of frequency. Free space is assumed to exist on either side of the medium. The functions $\varepsilon(z)$ and $\sigma(z)$ are to be piecewise continuously differentiable and, hence, may have jump discontinuities. If a layer is defined to be an interval in the medium on which ε and σ are continuously differentiable, then the program HATS is capable of handling a five-layer medium. A two-layer medium is pictured in Figure B-1.

The incident field can be of any form the user desires with the following two provisions:

1. The incident field must be a continuous function.

2. The incident field does not contact the medium with time t=0.
   Thus, transient as well as steady state fields can be determined by this program.

See Figure B-2 for a picture of some typical incident fields.

Details regarding proper input into the program and form of the output will be given in the following sections.

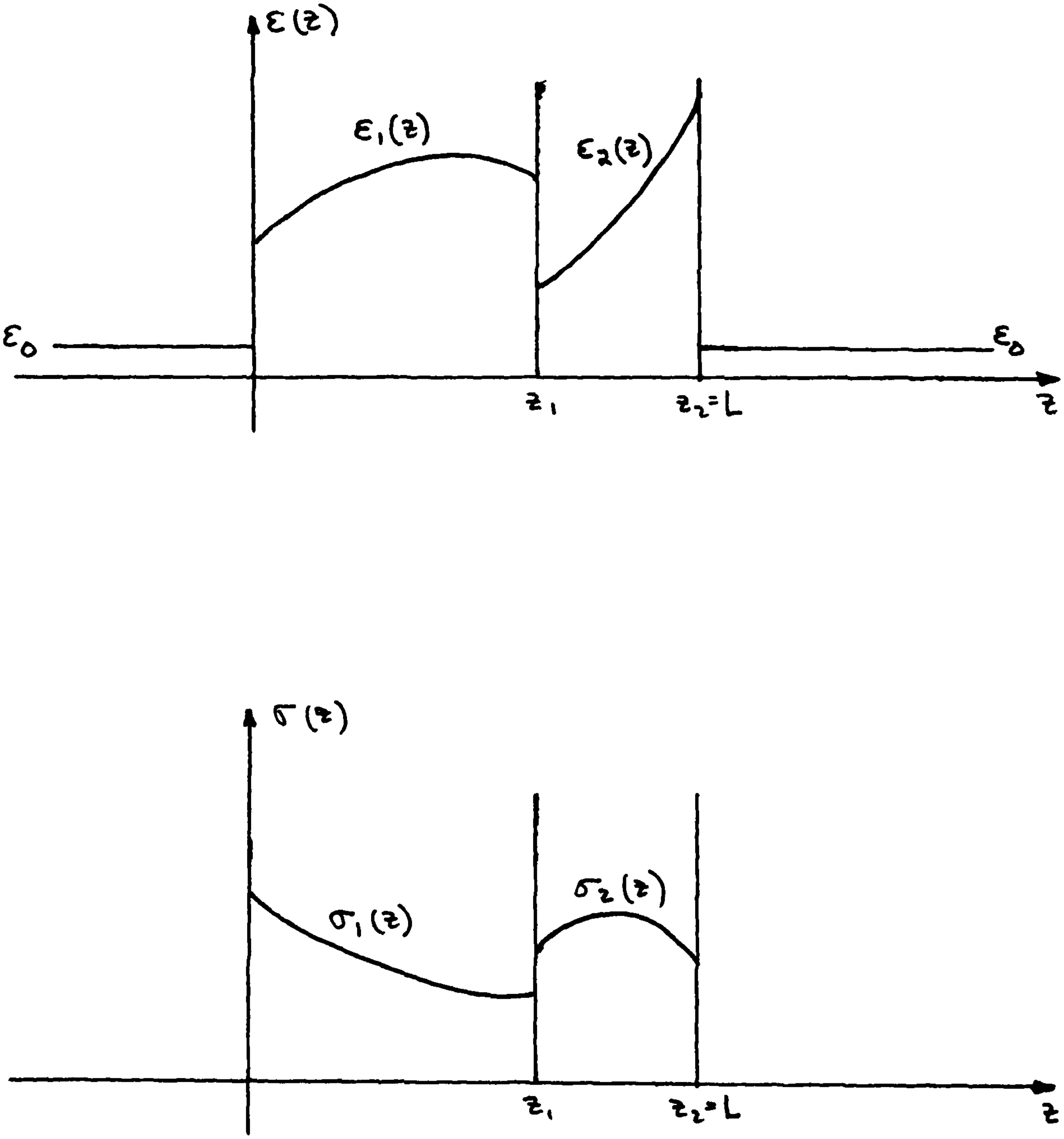

Figure B-1. Permittivity and conductivity profiles for a two-layer medium.

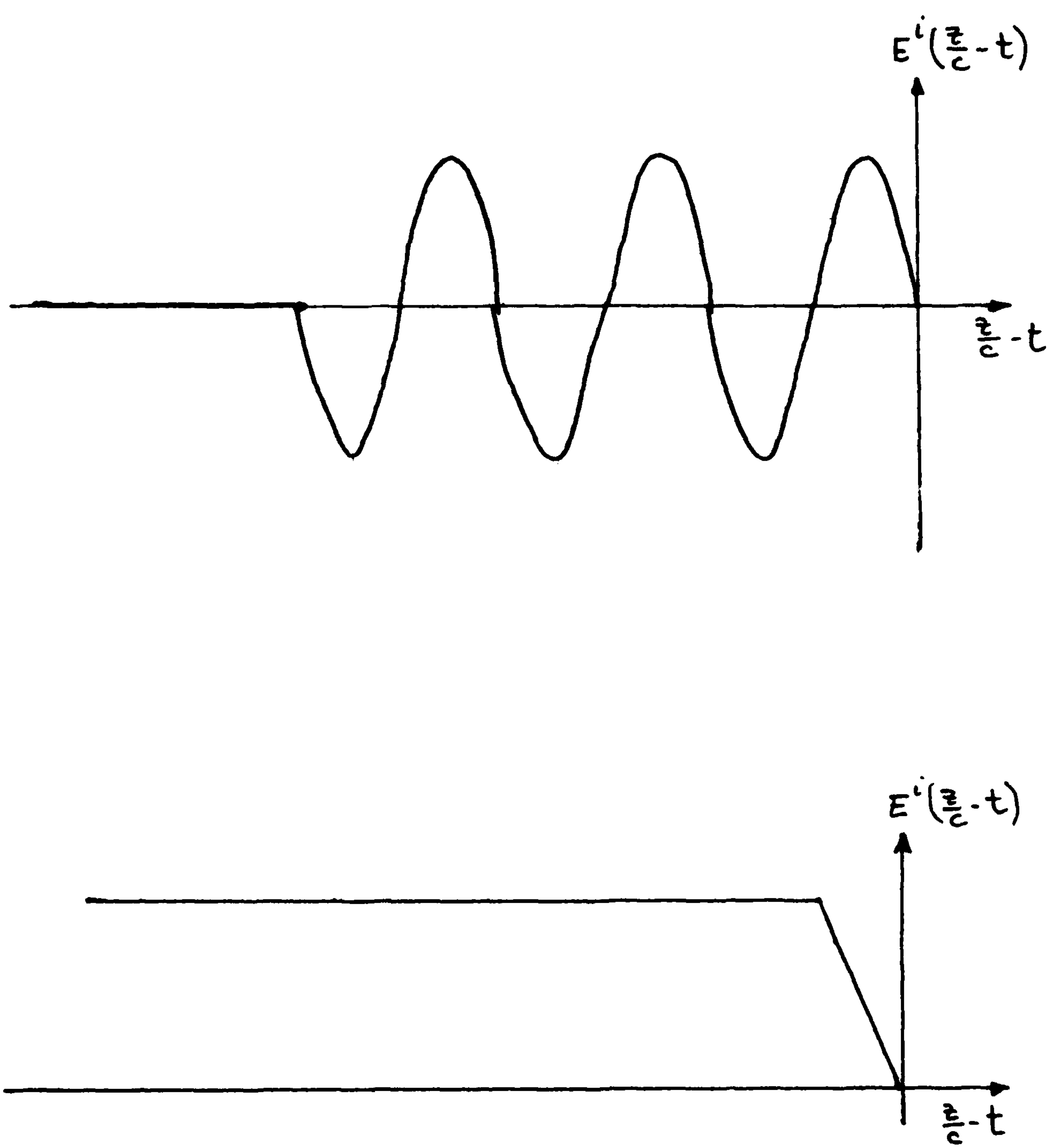

Figure B-2. Some typical incident fields, $E^i(\frac{z}{c} - t)$, where c is the speed of light in free space.

II. INPUTTING ε AND σ

The program presumes that a Liouville transform has been made on the spatial variable z and that a normalization has been performed on the time variable t. Therefore, a certain amount of massaging must be done before data and functions are put into the program, and some interpretation of the program results is necessary. All data are initially expressed in rationalized MKS units.

Consider a medium of NL layers, where $1 \le NL \le 5$. Layer i extends from $z=z_{i-1}$ to $z=z_i$, where $z_0=0$ and $z_{NL}=L$. Assume that the permittivity and conductivity are given on layer i by the functions $\varepsilon_i(z)$ and $\sigma_i(z)$, $z_{i-1}<z<z_i$, and let

$$\varepsilon(z)=\varepsilon_i(z) \text{ for } z_{i-1}<z<z_i,\ i=1,\ \cdots,\ NL.$$

The Liouville transform

$$x=x(z)=\left(\int_0^z\{\varepsilon(s)\mu_0\}^{\frac{1}{2}}ds\right)/\ell \tag{B.1}$$

where

$$\ell=\int_0^L\{\varepsilon(s)\mu_0\}^{\frac{1}{2}}ds$$

and the normalization

$$\tau=t/\ell$$

is used to produce a problem in (x,τ) coordinates in which the velocity of propagation is unity.

In the (x,τ) coordinate system the scattering medium extends from x=0 to x=1 and is again divided into NL layers. Properties of each of these layers (in terms of the x variable) are put into the program as function statements

appearing in subroutine EVAL. For each layer, four functions must be specified. For layer I ($1 \le I \le NL$) these functions are

$$AI(x)=-\ell\frac{d}{dz}\{\varepsilon_I(z)\mu_0\}^{-\frac{1}{2}},\ z_{I-1}<z<z_I$$

$$AIP(x)=\frac{d}{dx}AI(x)$$

$$BI(x)=-\ell\sigma_I(z)/\varepsilon_I(z),\ z_{I-1}<z<z_I$$

$$BIP(x)=\frac{d}{dx}BI(x).$$

These functions are to be entered in the appropriate lines in subroutine EVAL. If NL is less than five, the statement functions which appear in EVAL for layers NL+1, ···, 5 are not used. These unused statements may be left in the program.

As an example of how to determine these input functions, let NL=2, $z_0=0$, $z_1=.01$ m, $z_2=.03$ m, and

$$\varepsilon_1(z)=e_1\varepsilon_0$$

$$\varepsilon_2(z)=e_2\varepsilon_0$$

$$\sigma_1(z)=s_1 \text{ mho/m}$$

$$\sigma_2(z)=s_2 \text{ mho/m}$$

where e_1, e_2, s_1, s_2 are constants and ε_0 is the permittivity of free space,

$$\varepsilon_0=8.854\cdot10^{-12}\ \text{coul}^2/\text{nm}^2.$$

Then

$$\ell=\int_0^{.01}\{e_1\varepsilon_0\mu_0\}^{\frac{1}{2}}ds + \int_{.01}^{.03}\{e_2\varepsilon_0\mu_0\}^{\frac{1}{2}}ds$$

$$=(0.01\sqrt{e_1} + 0.02\sqrt{e_2})/c$$

where c is the speed of light in free space. It follows that for $0<z<0.01$ m,

$$x=x(z)=\left(z\sqrt{e_1}\right)/(\ell c)$$

and for $0.01<z<0.03$,

$$x-x(z)=\left(0.01\sqrt{e_1}+(z-0.01)\sqrt{e_2}\right)/(\ell c).$$

Thus,

$$A1(x)=0$$

$$A1P(x)=0$$

$$B1(x)=-\ell s_1/\left(e_1\varepsilon_0\right)$$

$$B1P(x)=0$$

$$A2(x)=0$$

$$A2P(x)=0$$

$$B2(x)=-\ell s_2/\left(e_2\varepsilon_0\right)$$

$$B2P(x)=0.$$

As another example, consider a one-layer medium of depth 1 m (i.e., $z_0=0$, $z_1=1$) with

$$\varepsilon(z)=9(z+2)^2\varepsilon_0,\ 0<z<1$$

$$\sigma(z)=\left(10-(z+1)^2\right)\cdot 10^{-3}\ \text{mho/m},\ 0<z<1.$$

Then

$$\ell=\frac{3}{c}\int_0^1(s+2)ds=15/2c,$$

$$x=x(z)=\left(z^2+4z\right)/5$$

and

$$z=z(x)=-2\sqrt{4+5x}.$$

It follows that

$$A1(x)=2.5/(z+2)^2=2.5/(4+5x)$$

$$A1P(x)=-12.5/(4+5x)^2$$

$$B1(x)=-10^{-3}(5+5x-2\sqrt{4+5x})/(6c\varepsilon_0\{4+5x\})$$

$$B1P(x)=-5\cdot 10^{-3}(4+5x-\sqrt{4+5x})/(6c\varepsilon_0\{4+5x\}^{5/2}).$$

For more general permittivity profiles, it may be necessary to solve equation (B.1) numerically for x as a function of z using, say, Newton's method. More will be said about this in Section VI.

III. INPUTTING THE INCIDENT WAVE

The "time" variable τ used in HATS has been normalized by setting

$$\tau=t/\ell$$

and hence, τ is a dimensionless quantity. Thus, if the incident field is $E^i(-t)$, $t>0$, then the corresponding incident field UINC to be used in the program is

$$UINC(-\tau)=E^i(-\tau\ell),\ \tau>0$$

or

$$UINC(\tau)=E^i(\tau\ell),\ \tau<0.$$

For example, consider an incident sinusoidal wave train of frequency f. Then

$$E^{i}(-t)=\sin(2\pi ft),\ t>0.$$

It is assumed that $E^{i}(-t)\equiv 0$ for $t<0$.) The incident field to be used in the program is

$$UINC(\tau)=\sin(-2\pi f\ell\tau),\ \tau<0.$$

This function is to be inserted into the subroutine INCWAV. Depending on the complexity of the incident field, a single function statement may not suffice to define the incident wave. This calls for an obvious modification of the subroutine. See the source listing in Section IX for an example of such a modification. In that example, a wave train of frequency f and of finite duration is generated.

IV. OTHER INPUT DATA

In addition to the function statements that must be written into the program, certain constants are to be read in as data.

Line 1: N, NL, IFRQ, ITM, ISCAT, IOPT

Format: (6I5)

N: number of subintervals to be used in the x interval [0,1]. Since HATS solves partial differential equations and integral equations numerically, the x interval [0,1] is split into N equal subintervals to establish a mesh of grid points. Thus, H=1/N is the step size used in most of the calculations. To reduce the effects of roundoff error, it is suggested (but not necessary) that N be a power of 2, say N=16, 32, 64, or 128. N must be an even integer not exceeding 128. Generally, the larger N is, the more accurate the program results, but this is at the expense of increased computation.

NL: number of layers in the medium. NL must be at least 1 and no greater than 5.

IFRQ: frequency of observation of the internal field. In addition to using a "spatial" (x) step size of H, HATS also uses a "temporal" (τ) step size of H. The internal field will be output at τ intervals of

$$\Delta\tau=2H\cdot IFRQ;$$

i.e., at time intervals of

$$\Delta\tau=2H\cdot IFRQ\cdot\ell.$$

ITM: maximum "time" to be considered. ITM is the maximum number of τ intervals of length 2H to be used in the program; i.e., the maximum time t considered will be

$$t=2H\cdot ITM\cdot\ell.$$

ITM must not exceed 2000 and should be an integral multiple of IFRQ.

ISCAT: output option. If ISCAT=1, the incident, reflected, and transmitted fields (i.e., field transmitted out the right-hand edge of the medium) will be printed out for $\tau=0$, 2H, 4H, $\cdots$, 2H$\cdot$ITM. If ISCAT=0, this printout is omitted.

IOPT: output option. If IOPT=0, the field within the medium will be printed out at the (1+N/2) grid points, x-0, 2H, 4H, $\cdots$, 1 for values of τ given by

$$\tau=2H\cdot IFRQ,\ 4H\cdot IFRQ,\ 6H\cdot IFRQ,\ \cdots,\ 2H\cdot ITM.$$

If IOPT=1, the field will be printed out for selected grid points (to be supplied later) at the above-mentioned τ values.

Line 2: X(1), X(2), ···, X(6)

Format: (6I5)

Assuming that layer i extends from $z=z_{i-1}$ to $z=z_i$, the Liouville transform (B.1) implies that in the x variable this layer extends from x_{i-1} to x_i, where

$$x_i=(\int_0^{z_i}\{\varepsilon(s)\mu_0\}^{\frac{1}{2}}ds)/\ell,\ i=0,\ 1,\ \cdots,\ NL.$$

The quantities X(i) are defined by

$$X(i)=x_{i-1}/H.$$

Notice that X(1)=0 and X(NL+1)=N. The X(i)'s are to be even integers satisfying

$$X(i)-X(i-1)\leq 32,\ i=2,3,\ \cdots,\ NL+1.$$

If the medium has fewer than five layers, the values assigned to X(NL+2), ···, X(6) are immaterial.

Line 3: C(1), C(2), ···, C(6)

Format: (6E12.5)

If the permittivity in layer i is given by $\varepsilon_i(z)$, then

$$C(i)=\{\varepsilon_i(z_i+)/\varepsilon_{i-1}(z_i-)\}^{\frac{1}{2}} \tag{B.2}$$

where, as usual, the z_i's denote the boundaries of the layers. For i=1, (B.2) is replaced by

$$C(1)=\{\varepsilon_1(0+)/\varepsilon_0\}^{\frac{1}{2}}$$

and for i=NL+1,

$$C(NL+1)=\{\varepsilon_0/\varepsilon_{NL}(L-)\}^{\frac{1}{2}}.$$

Again, if there are fewer than five layers, the values assigned to C(NL+2), •••, C(6) are immaterial.

The following data must also be supplied if IOPT=1.

Line 4: NOBPTS

Format: (I5)

If it is desired to determine the internal field only at selected spatial grid points, then NOBPTS is the number of points in the x interval [0,1] at which the field should be determined.

Line 5: IX

Format: (I5)

IX is an integer specifying the position of the first grid point at which the field should be determined. It is defined by

$$IX=x/H$$

where x is the point where the solution is desired. IX must be an even integer, $0 \leq IX \leq N$.

Line 6: IX

Format: (I5)

This specifies the position of the second point of observation.

This same input is continued until all NOBPTS positions have been specified.

V. OUTPUT

HATS automatically prints out the input data from lines 1, 2, and 3.

If ISCAT=1, the incident, reflected, and transmitted fields are printed next. The incident field is printed first in (1X,8E10.3) format for as many lines as is necessary to print the entire field. The array being printed out is

$$UI(J),\ J=1,\ 2,\ \cdots,\ ITM+1$$

where

$$UI(J)=UINC(-2(J-1)H)$$
$$=E^{i}(-2(J-1)H\ell)$$

The reflected field is printed next, using the same format. The array being printed here is

$$UR(J)=E^{r}(2(J-1)H\ell),\ J=1,\ 2,\ \cdots,\ ITM+1.$$

Finally the transmitted field is printed where

$$UT(J)=E^{t}(-2(J-1)H\ell),\ J=1,\ 2,\ \cdots,\ ITM+1.$$

The remaining output depends on the value of IOPT. If IOPT=0, the next line of output is an integer IT (in I10 format) followed by an array of (1+N/2) numbers in (1X,8E10.3) format. The array being printed out is

$$U(J)=u(2(J-1)H,\ 2\cdot IT\cdot H).$$

In other words, the field throughout the width of the medium is being printed for time $\tau=2\cdot IT\cdot H$. To relate this to the physical field E, note that

$$E(z,t)=u(x,\tau)$$

where, if $x_{i-1}\leq x\leq x_i$,

$$z=z_{i-1}+\left(\ell\int_{x_{i-1}}^{x}\exp\{-\int_{x_{i-1}}^{y}A(s)ds\}dy\right)/\left(\varepsilon_i(z_{i-1}+)\mu_0\right)^{\frac{1}{2}},$$

$$t=\tau\ell$$

and

$$A(s)=A_i(s) \text{ for } x_{i-1}\leq s\leq x_i.$$

This printout is continued for all values of IT given by

IT=IFTQ, 2•IFRQ, •••, ITM.

On the other hand, if IOPT=1, then the next line of output is an integer IX (in I10 format) followed by an array of 1+(ITM-(IX/2))/IFRQ numbers in (IX,8310.3) format. The array being printed is

U(J)=u(IX•H, 2(IFRQ•(J-1)+IX/2)H).

In other words, the field at position x=IX•H is being printed at τ intervals of 2H•IFRQ starting from τ=IX•H. This printout is continued for all NOBPTS values of IX specified by the user in the input data.

Finally, the message FINISHED is printed to indicate that the program has finished execution in a normal fashion.

In general, deducing any reasonable conclusion by looking at the printed output will be a formidable task. The user will probably find it much more

meaningful to plot the data, using, for example, a CALCOMP plotting routine. For this, a two-step process is suggested.

1. Run HATS, having the output transferred to a disk file.
2. Run the plotting routine, reading data from the disk file.

To facilitate step 1, all data is output from HATS via the subroutine PRNT. The comment statements in the subroutine are self-explanatory, indicating which blocks of code are used to output the various data. In addition, the commented write statements can be used to send a streamlined form of the output to a disk file. The user should note the format used so that the data is later read properly from the disk file. Also note that output device 8 needs to be defined via JCL.

An example of a plotting routine has been listed in Section X. The actual calls made to the graphing subroutines will have to be replaced with calls utilizing software at USAFSAM.

VI. PRECAUTIONS, TRICKS, AND GENERAL OBSERVATIONS

As with all numerical solutions, HATS is not infallible. Continuous but wildly varying conductivity and permittivity profiles within a layer will generally cause inaccurate solutions to be calculated. Also, if the conductivity is excessively large, the numerical solution of the partial differential equations will again prove unreliable. In this case internal and scattered fields will generally start to grow with time, so it will quickly become apparent to the user that the results are in error.

One method for trying to handle large conductivities or wildly varying conductivities and/or permittivities is to decrease step size H (i.e., increase N). Since this will generally cause the restriction

$$X(i)-X(i-1)\leq 32$$

to be violated, the following example shows how HATS can be "tricked."

Consider a two-layer medium and suppose N=32, with X(1)=0, X(2)=12, X(3)=32. If a smaller step size is needed and N should be 64, this will place the boundaries of the layers at X(1)=0, X(2)=24, X(3)=64; consequently

X(3)-X(2)=40>32.

To circumvent this problem, define the medium to consist of three layers instead of two, with boundaries, for example, at X(1)=0, X(2)=24, X(3)=44, X(4)=64. The function statements for A3(x), A3P(x), B3(x), B3P(x) will be exactly the same as those for layer 2. Further, C(1) and C(2) will be as in the N=32 case, and C(4) will have the value that C(3) has in the N=32 case. Finally, to indicate that there is indeed no discontinuity at X(3), set C(3)=1.0.

If N needs to be increased beyond 128, consult the last portion of this Section.

If a large step size (small N) is used, then detail in the solution is lost and some accuracy in the solution values calculated will generally be lost. On the other hand, fewer calculations will be performed within the program and the solution can be monitored over a longer period of time since the maximum value of τ is ITM•2H.

Extending the Maximum Time

If necessary, values of ITM greater than 2000 can be considered relatively easily, at the expense of increased storage. It suffices to increase the dimension of the arrays

V, BUFF, UI, U, UR

which appear in COMMON statements. Let ITM1=ITM+1. The new dimensions for the above-mentioned arrays are to be

V(5,ITM1,2), BUFF(200+ITM1), UI(ITM1), U(ITM1), UR(ITM1).

Incidentally, if there are fewer than five layers, storage can be reduced by changing the 5 in V(5,ITM1,2) to the actual number of layers. (In subroutine FIELD there is an array UT(2001) that contains the transmitted field. The dimension of this array need not be increased since UT is equivalenced to BUFF).

Increasing the Number of Intervals Allowed

If a value of NL greater than 5 is desired, it is necessary to

1. Change the dimension of all arrays in COMMON statements from 5 to the new value of NL (i.e., wherever a 5 appears, it should be replaced by NL).

2. Change the appropriate READ statements in the main program and WRITE statements in subroutine PRNT.

Decreasing the Step Size

If a value of N greater than 128 must be considered, it is necessary to

1. Change the dimension of all arrays appearing in COMMON statements from 129 to N+1.

2. Change the dimension of BUFF to N+ITM+3.

3. Change the EQUIVALENCE statement at the beginning of subroutine FIELD to

EQUIVALENCE (BUFF(N+2), UT(1)).

Numerical Solution of Equation (B.1)

For some permittivity profiles the Liouville transform (B.1) will need to be solved numerically for z as a function of x. This requires two new arrays to be generated, Z(I) and ZH(I), I=1, 2, ⋯, N, where

$$(I-1)H=\left(\int_0^{Z(I)}\{\varepsilon(s)\mu_0\}^{\frac{1}{2}}ds\right)/\ell$$

$$(I-.5)H=\left(\int_0^{ZH(I)}\{\varepsilon(s)\mu_0\}^{\frac{1}{2}}ds\right)/\ell$$

and Z(N+1)=L. The above equations are readily solved via Newton's method.

Since the functions AI, AIP, BI, BIP in subroutine EVAL cannot be expressed in terms of the x variable, they must now be written as functions of z,

$$AI(z)=-\ell\frac{d}{dz}\{\varepsilon_I(z)\mu_0\}^{-\frac{1}{2}},\ z_{I-1}<z<z_I$$

$$AIP(z)=\left(\frac{d}{dz}AI(z)\right)/\left(\frac{dx}{dz}\right)$$

$$BI(z)=-\ell\sigma_I(z)/\varepsilon_I(z),\ z_{I-1}<z<z_I$$

$$BIP(z)=\left(\ell\frac{d}{dz}BI(z)\right)/\{\varepsilon_I(z)\mu_0\}^{\frac{1}{2}}.$$

Finally, in subroutine EVAL the statements

```
XR=(I-1)*H
XR=(I-.5)*H
```

should be replaced with

```
XR=Z(I)
XR=ZH(I)
```

respectively, and the remaining eight statements of the form

```
XR=(X(J)+I-1)*H
XR=(X(J)+I-.5)*H
```

should be replaced with

```
XR=Z(NPTL(J-1)+I-1)
XR=ZH(NPTL(J-1)+I-1).
```

VII. SOURCE LISTING OF HATS

```
C..........MAIN PROGRAM
      INTEGER X
      COMMON /BLOCK1/H,NL,NTGP,NPTL(5),IFRQ,ITM,IT,ISCAT,IOPT
      COMMON /BLOCK2/A(33),B(33),AP(33),BP(33),DT(33),AH(33),BH(33),
     1               APH(33),BPH(33),DTH(33),AI(33),BI(33),DTI(33),
     2               DP(33),EBI(33),AR(5),AL(5),BR(5),BL(5)
      COMMON /BLOCK3/X(6),C(6),DPI(33),DPBI(33),EAMB(5,33),EAPB(5,33),
     1               QOLD(33),QNEW(33),XP(5,129),XM(5,129),YP(5,129),
     2               YM(5,129),XPC(5),XMC(5),YPC(5),YMC(5),APBI(5),
     3               AMBI(5),P(5,2,33,33)
      COMMON /BLOCK4/JV(5,16),JW(5,16),SV(5,16),SW(5,16),V(5,2001,2),
     1               W(5,129,2),VS(129,2),WS(129,2)
      COMMON /BLOCK5/BUFF(2201),UI(2001),U(2001),UR(2001)
      READ (5,2000)  N, NL, IFRQ, ITM, ISCAT, IOPT, (X(I),I = 1,6)
      READ (5,3000)  (C(I),I = 1,6)
      NTGP = N + 1
      H = 1.0 / N
      CALL PRNT(1)
      CALL KERNEL
      CALL DELTA
      CALL INCWAV
      CALL FIELD
      CALL PRNT(6)
      STOP
 2000 FORMAT (6I5,/,6I5)
 3000 FORMAT (6E12.5)
      END
```

```
      SUBROUTINE EVAL(LAYER,ICK)
C..........INSERT FUNCTION STATEMENTS FOR EACH LAYER
      A1(X) = 0.0  + 0. * X
      B1(X) = 0.0 + 0. * X
      A1P(X) = 0.0 + 0. * X
      B1P(X) = 0.0 + 0. * X
      A2(X) = 0.0  + 0. * X
      B2(X) = 0.0 + 0. * X
      A2P(X) = 0.0 + 0. * X
      B2P(X) = 0.0 + 0. * X
      A3(X) = 0.0  + 0. * X
      B3(X) = 0.0 + 0. * X
      A3P(X) = 0.0 + 0. * X
      B3P(X) = 0.0 + 0. * X
      A4(X) = 0.0  + 0. * X
      B4(X) = 0.0 + 0. * X
      A4P(X) = 0.0 + 0. * X
      B4P(X) = 0.0 + 0. * X
      A5(X) = 0.0  + 0. * X
      B5(X) = 0.0 + 0. * X
      A5P(X) = 0.0 + 0. * X
      B5P(X) = 0.0 + 0. * X
C..........
      INTEGER X
      COMMON /BLOCK1/H,NL,NTGP,NPTL(5),IFRQ,ITM,IT,ISCAT,IOPT
      COMMON /BLOCK2/A(33),B(33),AP(33),BP(33),DT(33),AH(33),BH(33),
     1              APH(33),BPH(33),DTH(23),AI(33),BI(33),DTI(33),
     2              DP(33),EBI(33),AR(5),AL(5),BR(5),BL(5)
```

```
      COMMON /BLOCK3/X(6),C(6),DPI(33),DPBI(33),EAMB(5,33),EAPB(5,33),
     1              QOLD(33),QNEW(33),XP(5,129),XM(5,129),YP(5,129),
     2              YM(5,129),XPC(5),XMC(5),YPC(5),YMC(5),APBI(5),
     3              AMBI(5),P(5,2,33,33)
      HS = H / 6.
      NP1 = NPTL(LAYER)
      IF (ICK.EQ.2)   GO TO 70
      IF (LAYER - 2)   10, 20, 30
   10 DO 15  I = 1, NP1
      XR = (I - 1.) * H
      A(I) = A1(XR)
      B(I) = B1(XR)
      AP(I) = A1P(XR)
      BP(I) = B1P(XR)
      DT(I) = (B(I) ** 2 - A(I) ** 2) / 4.0
      XR = (I - .5) * H
      AH(I) = A1(XR)
      BH(I) = B1(XR)
      APH(I) = A1P(XR)
      BPH(I) = B1P(XR)
      DTH(I) = (BH(I) ** 2 - AH(I) ** 2) / 4.0
   15 CONTINUE
      GO TO 67
   20 DO 25  I = 1, NP1
      XR = (X(2) + I - 1.) * H
      A(I) = A2(XR)
      B(I) = B2(XR)
      AP(I) = A2P(XR)
      BP(I) = B2P(XR)
      DT(I) = (B(I) ** 2 - A(I) ** 2) / 4.0
      XR = (X(2) + I - .5) * H
      AH(I) = A2(XR)
      BH(I) = B2(XR)
      APH(I) = A2P(XR)
      BPH(I) = B2P(XR)
      DTH(I) = (BH(I) ** 2 - AH(I) ** 2) / 4.0
   25 CONTINUE
      GO TO 67
   30 IF (LAYER - 4)   40, 50, 60
   40 DO 45  I = 1, NP1
      XR = (X(3) + I - 1.) * H
      A(I) = A3(XR)
      B(I) = B3(XR)
      AP(I) = A3P(XR)
      BP(I) = B3P(XR)
      DT(I) = (B(I) ** 2 - A(I) ** 2) / 4.0
      XR = (X(3) + I - .5) * H
      AH(I) = A3(XR)
      BH(I) = B3(XR)
      APH(I) = A3P(XR)
      BPH(I) = B3P(XR)
      DTH(I) = (BH(I) ** 2 - AH(I) ** 2) / 4.0
   45 CONTINUE
      GO TO 67
   50 DO 55  I = 1, NP1
      XR = (X(4) + I - 1.) * H
      A(I) = A4(XR)
      B(I) = B4(XR)
      AP(I) = A4P(XR)
      BP(I) = B4P(XR)
```

```
      DT(I) = (B(I) ** 2 - A(I) ** 2) / 4.0
      XR = (X(4) + I - .5) * H
      AH(I) = A4(XR)
      BH(I) = B4(XR)
      APH(I) = A4P(XR)
      BPH(I) = B4P(XR)
      DTH(I) = (BH(I) ** 2 - AH(I) ** 2) / 4.0
   55 CONTINUE
      GO TO 67
   60 DO 65  I = 1, NP1
      XR = (X(5) + I - 1.) * H
      A(I) = A5(XR)
      B(I) = B5(XR)
      AP(I) = A5P(XR)
      BP(I) = B5P(XR)
      DT(I) = (B(I) ** 2 - A(I) ** 2) / 4.0
      XR = (X(5) + I - .5) * H
      AH(I) = A5(XR)
      BH(I) = B5(XR)
      APH(I) = A5P(XR)
      BPH(I) = B5P(XR)
      DTH(I) = (BH(I) ** 2 - AH(I) ** 2) / 4.0
   65 CONTINUE
   67 CONTINUE
      AI(NP1) = 0.0
      BI(NP1) = 0.0
      DTI(NP1) = 0.0
      EBI(NP1) = 1.0
      N = NP1 - 1
      DO 68  K = 1, N
      J = NP1 - K
      JP = J + 1
      AI(J) = AI(JP) + HS * (A(J) + 4. * AH(J) + A(JP))
      BI(J) = BI(JP) + HS * (B(J) + 4. * BH(J) + B(JP))
      DTI(J) = DTI(JP) + HS * (DT(J) + 4. * DTH(J) + DT(JP))
      EBI(J) = EXP( BI(J) )
   68 CONTINUE
      GO TO 90
   70 DO 80   I = 1, NP1
      B(I) = - B(I)
      BP(I) = - BP(I)
      BH(I) = - BH(I)
      BPH(I) = - BPH(I)
      BI(I) = - BI(I)
      EBI(I) = 1.0 / EBI(I)
   80 CONTINUE
   90 CONTINUE
      AR(LAYER) = A(NP1)
      BR(LAYER) = B(NP1)
      AL(LAYER) = A(1)
      BL(LAYER) = B(1)
      DO 100  I = 1, NP1
      DP(I) = DT(I) - (AP(I) - BP(I)) / 2.
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INCWAV
C..........INSERT FUNCTION STATEMENT FOR INCIDENT FIELD
      UINC(X) = 0.0 + 0. * X
C..........
      COMMON /BLOCK1/H,NL,NTGP,NPTL(5),IFRQ,ITM,IT,ISCAT,IOPT
      COMMON /BLOCK5/BUFF(2201),UI(2001),U(2001),UR(2001)
      H2 = H * 2.
      ITM1 = ITM + 1
      DO 10  J = 1, ITM1
      X = - (J - 1) * H2
      UI(J) = UINC(X)
   10 CONTINUE
      RETURN
      END


      SUBROUTINE PRNT(K)
      INTEGER X
      COMMON /BLOCK1/H,NL,NTGP,NPTL(5),IFRQ,ITM,IT,ISCAT,IOPT
      COMMON /BLOCK2/A(33),B(33),AP(33),BP(33),DT(33),AH(33),BH(33),
     1               APH(33),BPH(33),DTH(33),AI(33),BI(33),DTI(33),
     2               DP(33),EBI(33),AR(5),AL(5),BR(5),BL(5)
      COMMON /BLOCK3/X(6),C(6),DPI(33),DPBI(33),EAMB(5,33),EAPB(5,33),
     1               QOLD(33),QNEW(33),XP(5,129),XM(5,129),YP(5,129),
     2               YM(5,129),XPC(5),XMC(5),YPC(5),YMC(5),APBI(5),
     3               AMBI(5),P(5,2,33,33)
      COMMON /BLOCK4/JV(5,16),JW(5,16),SV(5,16),SW(5,16),V(5,2001,2),
     1               W(5,129,2),VS(129,2),WS(129,2)
      COMMON /BLOCK5/BUFF(2201),UI(2001),U(2001),UR(2001)
      IF (K - 2)  1, 2, 50
    1 CONTINUE
C..........PRINT OUT INPUT DATA FROM LINES 1, 2, 3
      N = NTGP -1
      NLL = NL + 1
      WRITE (6,2000)  N, NL, IFRQ, ITM, ISCAT, IOPT, (X(I), I = 1, NLL)
      WRITE (6,3000)  (C(I),I = 1, NLL)
C      WRITE (8,2000)  N, NL, IFRQ, ITM
      RETURN
    2 CONTINUE
      RETURN
   50 CONTINUE
      IF (K - 4)  3, 4, 60
    3 CONTINUE
C..........PRINT INCIDENT, REFLECTED AND TRANSMITTED FIELDS IF ISCAT = 1
      IF (ISCAT .EQ. 0)  RETURN
      ITM1 = ITM + 1
      WRITE (6,5000)
      WRITE (6,1000)  (UI(I),I = 1, ITM1)
      WRITE (6,5100)
      WRITE (6,1000)  (UR(I),I = 1, ITM1)
      WRITE (6,5200)
      WRITE (6,1000)  (V(NL,I,1),I = 1, ITM1)
C      WRITE (8,1500)  (V(NL,I,1),I = 1, ITM1)
C      WRITE (8,1500)  (UR(I),I = 1, ITM1)
      RETURN
    4 CONTINUE
C..........PRINT FIELD AT ALL GRID POINTS INSIDE MEDIUM AT TIME TAU = IT
      WRITE (6,7000)  IT
      WRITE (6,1000)  (U(I),I = 1, NTGP, 2)
```

```
C        WRITE (8,6000)   IT
C        WRITE (8,1500)   (U(I),I = 1, NTGP, 2)
         RETURN
      60 CONTINUE
         IF (K - 6)   5, 6, 6
       5 CONTINUE
C..........PRINT FIELD AT POSITION X = IT * H   FOR TIMES
C............TAU = IT * H, (IT + IFRQ * 2) * H, ...
         IMAX = 1 + (ITM - (IT / 2) ) / IFRQ
         WRITE (6,8000)   IT
         WRITE (6,1000)   (U(I),I = 1, IMAX)
C        WRITE (8,6000)   IT
C        WRITE (8,1500)   (U(I),I = 1, IMAX)
         RETURN
       6 CONTINUE
         WRITE (6,4000)
         RETURN
    1000 FORMAT (1X,8E10.3)
    1500 FORMAT (8E10.3)
    2000 FORMAT (6I5,/,6I5)
    3000 FORMAT (6E12.5)
    4000 FORMAT (///,1X,8HFINISHED)
    5000 FORMAT (///,1X,14HINCIDENT FIELD,/)
    5100 FORMAT (///,1X,15HREFLECTED FIELD,/)
    5200 FORMAT (///,1X,17HTRANSMITTED FIELD,/)
    6000 FORMAT (I10)
    7000 FORMAT (///,1X,29HINTERNAL FIELD AT TIME TAU = ,I10,8H * 2 * H,/)
    8000 FORMAT (///,1X,21HFIELD AT POSITION X =,I10,4H * H,/)
         END


         SUBROUTINE KERNEL
C..........THIS SUBROUTINE SOLVES SYSTEMS OF PARTIAL DIFFERENTIAL
C..............EQUATIONS TO OBTAIN RIEMANN FUNCTIONS FOR EACH LAYER
C..............OF THE MEDIUM
         INTEGER X
         COMMON /BLOCK1/H,NL,NTGP,NPTL(5),IFRQ,ITM,IT,ISCAT,IOPT
         COMMON /BLOCK2/A(33),B(33),AP(33),BP(33),DT(33),AH(33),BH(33),
        1                   APH(33),BPH(33),DTH(33),AI(33),BI(33),DTI(33),
        2                   DP(33),EBI(33),AR(5),AL(5),BR(5),BL(5)
         COMMON /BLOCK3/X(6),C(6),DPI(33),DPBI(33),EAMB(5,33),EAPB(5,33),
        1                   QOLD(33),QNEW(33),XR(5,129),XM(5,129),YP(5,129),
        2                   YM(5,129),XPC(5),XMC(5),YPC(5),YMC(5),APBI(5),
        3                   AMBI(5),P(5,2,33,33)
         DIMENSION W(33)
C..........NL = NUMBER OF LAYERS
C..........NPTL(I) = NUMBER OF GRID POINTS IN LAYER I
         DO 10  I = 1, NL
         NPTL(I) = X(I+1) - X(I) + 1
      10 CONTINUE
         H4 = H/4
         H6 = H/6
         DO 110   I = 1, NL
         F1 = - (C(I+1) + 1.) / 2.0
         F2 =   (C(I+1) + 1.) / 4.0
         F3 =   (C(I+1) - 1.) / 2.0
         F4 = F3 / 4.0
         F5 = F3 / 2.0
         NP1 = NPTL(I)
```

```
      N = NP1 - 1
      DO 100  ICK = 1,2
      CALL EVAL(I,ICK)
      F6 = (AR(I) - BR(I)) / 4.0
      D1 = F3 * (AR(I) + BR(I)) / 2.0
      D2 = - F2 * F6 * 4.0
      D3 = F2 * DP(NP1)
      DPI(NP1) = 0.0
      DPBI(NP1) = 0.0
      AMB = (AR(I) - BR(I)) / 2.0
      DO 20  K = 1, N
      J = NP1 - K
      JP = J + 1
      DPI(J) = DTI(J) + (A(J) - B(J)) / 2.0 - AMB
      DPBI(J) = DPBI(JP) + HS * (B(J) * (DT(J) - (AP(J) - BP(J)) / 2.0)
     1            + 4. * BH(J) * (DTH(J) - (APH(J) - BPH(J)) / 2.0)
     2            + B(JP) * (DT(JP) - (AP(JP) - BP(JP)) / 2.0))
   20 CONTINUE
C..........COMPUTE BOUNDARY VALUES
      DO 25  K = 1, NP1
      P(I,ICK,K,1) = F1 * (DTI(K) + (A(K) - B(K)) / 2.0) + D1
      J = NP1 + 1 - K
      P(I,ICK,NP1,K) = EBI(J) * (F3 * (DTI(J) + (A(J) + B(J))/2.0) + D2)
   25 CONTINUE
C..........GENERATE SOLUTION OF PDE
      DO 40 NCT = 2, N
      LN = N + 2 - NCT
      KMAX = NCT - 1
      DN = DP(LN)
      DNM = DP(LN-1)
      EN = EBI(LN)
      QOLD(1) = F2 * DN/EN
      ENM = EBI(LN-1)
      QOLD(NCT) = D3 + (F4 * DPI(LN) - F6)*DPI(LN) + F5 * DPBI(LN)
      EQ = ENM/EN
      H4 = H/4.
      HE = H/EQ
      DE = DN/EN
      HDE = H4 * DN * EQ
      DET = 1. + H4 * H * DNM
      DO 30  J = 1, KMAX
      M = LN + J
      M1 = M - 1
      J1 = J + 1
      P(I,ICK,M1,J1) = (P(I,ICK,M,J1) - H * (HDE * P(I,ICK,M1,J) +
     1             EN * QOLD(J1) + ENM * QOLD(J)))/DET
      QNEW(J1) = (QOLD(J) + H4 * (DE * P(I,ICK,M1,J) +
     1             DNM * (P(I,ICK,M,J1)/ENM - HE *QOLD(J1))))/DET
   30 CONTINUE
      DO 35  J = 2,NCT
      QOLD(J) = QNEW(J)
   35 CONTINUE
   40 CONTINUE
      QNEW(1) = F2 * DNM/ENM
      QNEW(NP1) = D3 + (F4 * DPI(1) - F6) * DPI(1) + F5 * DPBI(1)
C.......COMPUTE INTEGRAL FOR SCATTERING KERNEL
      F7 = EBI(1) * 2.
      F8 = (AL(I) - BL(I)) / 2.0
      W(1) = 0.
```

```
      W1 = F7 *QNEW(1) - F8 * P(I.ICK.1.1)
      DO 50  K = 2. NP1
      W2 = F7 *QNEW(K) - F8 * P(I.ICK.K.K)
      W(K) = W(K-1) + H * (W1 + W2)
      W1 = W2
   50 CONTINUE
      IF (ICK.EQ.2)  GO TO 70
      F9 = EBI(1)
      F11 = F8 * (C(I+1) + 1.)
      DO 60  K = 1. NP1
      XP(I.K) = ( -F11 + W(K)) / EBI(1)
      XM(I.K) = F11 + 2. * P(I.1.K.K) - W(K)
   60 CONTINUE
      XMC(I) = XM(I.NP1) - F3 * (AL(I) + BL(I)) * EBI(1)
      XPC(I) = - XMC(I) / EBI(1)
      GO TO 90
   70 G1 = F3 * (AL(I) + BL(I)) * EBI(1)
      G2 = -F1 * (AL(I) - BL(I))
      DO 80  KT = 1. NP1
      K = NP1 - KT + 1
      YP(I.KT) = G1 - 2.* P(I.2.NP1.NP1) + 2.* P(I.2.K.K) + W(NP1)- W(K)
      YM(I.KT) = ( -G1 + 2.* P(I.2.NP1.NP1) - W(NP1) + W(K)) / EBI(1)
   80 CONTINUE
      YPC(I) = G1 - 2.* P(I.2.NP1.NP1) - G2 + W(NP1)
      YMC(I) = - YPC(I) / EBI(1)
   90 CONTINUE
  100 CONTINUE
      DO 105  J = 1. NP1
      EAMB(I.J) = EXP( (AI(J) + BI(J)) / 2. )  /  2.0
      EAPB(I.J) = EXP( (AI(J) - BI(J)) / 2. )  /  2.0
  105 CONTINUE
      BR(I) = - BR(I)
      BL(I) = - BL(I)
      APBI(I) = 2. * EAPB(I.1)
      AMBI(I) = 2. * EAMB(I.1)
  110 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DELTA
C..........THIS SUBROUTINE COMBINES THE RIEMANN FUNCTIONS FROM
C..............SUBROUTINE KERNEL TO FORM THE SCATTERING KERNELS
      INTEGER X
      REAL*8 SUM1,SUM2,SUM3,SUM4
      COMMON /BLOCK1/H,NL,NTGP,NPTL(5),IFRQ,ITM,IT,ISCAT,IOPT
      COMMON /BLOCK3/X(6),C(6),DPI(33),DPBI(33),EAMB(5,33),EAPB(5,33),
     1               QOLD(33),QNEW(33),XP(5,129),XM(5,129),YP(5,129),
     2               YM(5,129),XPC(5),XMC(5),YPC(5),YMC(5),APBI(5),
     3               AMBI(5),P(5,2,33,33)
      COMMON /BLOCK4/JV(5,16),JW(5,16),SV(5,16),SW(5,16),V(5,2001,2),
     1               W(5,129,2),VS(129,2),WS(129,2)
      N = NL
      NTGP = 0
      DO 10  I = 1, NL
      NTGP = NTGP + NPTL(I)
   10 CONTINUE
      NTGP = NTGP - NL + 1
      NTGP1 = NTGP - 1
      DO 30  I = 1, NL
      NP2 = NPTL(I) + 1
      DO 20  K = NP2, NTGP
      XP(I,K) = XPC(I)
      XM(I,K) = XMC(I)
      YP(I,K) = YPC(I)
      YM(I,K) = YMC(I)
   20 CONTINUE
   30 CONTINUE
C.......CALCULATE LOCATION AND STRENGTH OF SINGULARITIES
      NM1 = N - 1
      JV(N,1) = 0
      JW(N,1) = 2 * X(N + 1)
      SV(N,1) = AMBI(N) * (C(N + 1) + 1.) / 2.
      SW(N,1) = - APBI(N) * (C(N + 1) - 1.) / 2.
      IF (N.EQ.1)  GO TO 70
      DO 60  IC = 1, NM1
      NCL = N - IC
      NCK = NCL + 1
      NJ = 2 ** (N - NCL)
      NJ2 = NJ / 2
      DO 40  I = 1, NJ2
      JV(NCL,I) = JV(NCK,I)
      JW(NCL,I) = JW(NCK,I)
      SV(NCL,I) = AMBI(NCL) * (C(NCK) + 1.) * SV(NCK,I) / 2.
      SW(NCL,I) = APBI(NCL) * (C(NCK) + 1.) * SW(NCK,I) / 2.
   40 CONTINUE
      NJ3 = NJ2 + 1
      DO 50  I = NJ3, NJ
      JV(NCL,I) = 2 * X(NCK) - JW(NCK,I - NJ2)
      JW(NCL,I) = 2 * X(NCK) - JV(NCK,I - NJ2)
      SV(NCL,I) = - AMBI(NCL) * (C(NCK) - 1.) * SW(NCK,I - NJ2) / 2.
      SW(NCL,I) = - APBI(NCL) * (C(NCK) - 1.) * SV(NCK,I - NJ2) / 2.
   50 CONTINUE
   60 CONTINUE
   70 CONTINUE
      AP = APBI(N) / 4.0
      AM = AMBI(N) / 4.0
      DO 80  K = 1, NTGP
      V(N,K,1) = AM * XM(N,K)
      V(N,K,2) = V(N,K,1)
```

```
            W(N,K,1) = AP * XP(N,K)
            W(N,K,2) = W(N,K,1)
   80    CONTINUE
            V(N,1,1) = 0.
            W(N,1,1) = 0.
            NP1 = NPTL(N)
            V(N,NP1,2) = AM * XMC(N)
            W(N,NP1,2) = AP * XPC(N)
            IF (N.EQ.1)  GO TO 240
            DO 230  IC = 1, NM1
            I = N - IC
C.......CALCULATE INTEGRALS FOR LAYERS N-1,N-2,...,2,1, IGNORING SINGULARIT
            NP1 = NPTL(I)
            N1 = NP1 - 1
            AM = AMBI(I) / 4.
            AP = APBI(I) / 4.
            IP = I + 1
            CM = 2. * (C(IP) - 1.)
            CP = 2. * (C(IP) + 1.)
            V(I,1,1) = 0.
            V(I,1,2) = AM * (CP * V(IP,1,2) - CM * W(IP,1,2))
            W(I,1,1) = 0.
            W(I,1,2) = 0.
            DO 110  K = 1, N1
            SUM1 = 0.D0
            SUM2 = 0.D0
            DO 90  J = 1, K
            IK = K - J + 2
            JP = J + 1
            IL = IK - 1
            SUM1 = SUM1 + XM(I,J) * V(IP,IK,1) + XM(I,JP) * V(IP,IL,2)
          1             + YM(I,J) * W(IP,IK,1) + YM(I,JP) * W(IP,IL,2)
            SUM2 = SUM2 + XP(I,J) * V(IP,IK,1) + XP(I,JP) * V(IP,IL,2)
          1             + YP(I,J) * W(IP,IK,1) + YP(I,JP) * W(IP,IL,2)
   90    CONTINUE
            KP = K + 1
            DO 100  M = 1, 2
            V(I,KP,M) = AM * (CP * V(IP,KP,M) - CM * W(IP,KP,M) + H * SUM1)
            W(I,KP,M) = AP * H * SUM2
  100    CONTINUE
  110    CONTINUE
            W(I,NP1,2) = W(I,NP1,2) + AP * (CP * W(IP,1,2) - CM * V(IP,1,2))
            SUM3 = 0.D0
            SUM4 = 0.D0
            DO 140  K = NP1, NTGP1
            SUM1 = 0.D0
            SUM2 = 0.D0
            DO 120  J = 1, N1
            IK = K - J + 2
            JP = J + 1
            IL = IK - 1
            SUM1 = SUM1 + XM(I,J) * V(IP,IK,1) + XM(I,JP) * V(IP,IL,2)
          1             + YM(I,J) * W(IP,IK,1) + YM(I,JP) * W(IP,IL,2)
            SUM2 = SUM2 + XP(I,J) * V(IP,IK,1) + XP(I,JP) * V(IP,IL,2)
          1             + YP(I,J) * W(IP,IK,1) + YP(I,JP) * W(IP,IL,2)
  120    CONTINUE
            SUM3 = SUM3 + V(IP,K - NP1 + 2,1) + V(IP,K - NP1 + 1,2)
            SUM4 = SUM4 + W(IP,K - NP1 + 2,1) + W(IP,K - NP1 + 1,2)
            KP = K + 1
            KM = K - NP1 + 2
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```
            DO 130  M = 1, 2
            V(I.KP.M) = AM * (CP * V(IP.KP.M) - CM * W(IP.KP.M)
           1                 + H * (SUM1 + XMC(I) * SUM3 + YMC(I) * SUM4))
            W(I.KP.M) = AP * (CP * W(IP.KM.M) - CM * V(IP.KM.M)
           1                 + H * (SUM2 + XPC(I) * SUM3 + YPC(I) * SUM4))
        130 CONTINUE
        140 CONTINUE
      C.......ADD IN CONTRIBUTIONS DUE TO SINGULARITIES
            NJ = 2 ** (N - I - 1)
            DO 220  K = 1, NJ
            JVK = JV(I + 1.K) / 2
            JWK = JW(I + 1.K) / 2
            SVK = SV(I + 1.K)
            SWK = SW(I + 1.K)
            V(I.1 - JVK.2) = V(I.1 - JVK.2) + AM * SVK * XM(I.1)
            W(I.1 - JVK.2) = W(I.1 - JVK.2) + AP * SVK * XP(I.1)
            NS = NTGP - 1 + JVK
            NF = 2 - JVK
            DO 160  KI = NF, NTGP
            DO 150  M = 1, 2
            V(I.KI.M) = V(I.KI.M) + AM * SVK * XM(I.KI + JVK)
            W(I.KI.M) = W(I.KI.M) + AP * SVK * XP(I.KI + JVK)
        150 CONTINUE
        160 CONTINUE
            IF (NS - N1)  180, 170, 170
        170 KKI = NP1 - JVK
            V(I.NP1 - JVK.2) = (V(I.NP1 - JVK.2) - AM * SVK * XM(I.NP1))
           1                     + AM * SVK * XMC(I)
            W(I.NP1 - JVK.2) = (W(I.NP1 - JVK.2) - AP * SVK * XP(I.NP1))
           1                     + AP * SVK * XPC(I)
        180 NS = NTGP - 1 + X(I + 1) - JWK
            NF = JWK - X(I + 1) + 2
            V(I.NF - 1.2) = V(I.NF - 1.2) + AM * SWK * YM(I.1)
            W(I.NF - 1.2) = W(I.NF - 1.2) + AP * SWK * YP(I.1)
            DO 200  KI = NF, NTGP
            KKI = KI + X(I + 1) - JWK
            DO 190  M = 1, 2
            V(I.KI.M) = V(I.KI.M) + AM * SWK * YM(I.KKI)
            W(I.KI.M) = W(I.KI.M) + AP * SWK * YP(I.KKI)
        190 CONTINUE
        200 CONTINUE
            IF (NS - N1)  220, 210, 210
        210 KKI = JWK - X(I) + 1
            V(I.KKI.2) =(V(I.KKI.2) - AM*SWK*YM(I.NP1))+ AM*SWK*YMC(I)
            W(I.KKI.2) =(W(I.KKI.2) - AP*SWK*YP(I.NP1)) +AP*SWK*YPC(I)
        220 CONTINUE
        230 CONTINUE
        240 CONTINUE
      C.......CALCULATE SCATTERING KERNELS WS AND VS
            CP = (C(1) + 1.) / 2.
            CM = (C(1) - 1.) / 2.
            DO 260  K = 1, NTGP
            DO 250  M = 1, 2
            VS(K.M) = - CM * V(1.K.M) + CP * W(1.K.M)
            WS(K.M) =   CP * V(1.K.M) - CM * W(1.K.M)
        250 CONTINUE
        260 CONTINUE
            CALL PRNT(2)
            RETURN
            END
```

```
      SUBROUTINE FIELD
C..........THIS SUBROUTINE COMPUTES SCATTERED AND INTERNAL FIELDS
C..............THROUGH USE OF THE SCATTERING KERNELS OBTAINED IN
C..............SUBROUTINE DELTA AND THE RIEMANN FUNCTIONS OBTAINED
C..............IN SUBROUTINE KERNEL
      INTEGER X
      DIMENSION UT(2001),WW(129),S(32),JJ(32)
      COMMON /BLOCK1/H,NL,NTGP,NPTL(5),IFRQ,ITM,IT,ISCAT,IOPT
      COMMON /BLOCK2/A(33),B(33),AP(33),BP(33),DT(33),AH(33),BH(33),
     1               APH(33),BPH(33),DTH(33),AI(33),BI(33),DTI(33),
     2               DP(33),EBI(33),AR(5),AL(5),BR(5),BL(5)
      COMMON /BLOCK3/X(6),C(6),DPI(33),DPBI(33),EAMB(5,33),EAPB(5,33),
     1               QOLD(33),QNEW(33),XP(5,129),XM(5,129),YP(5,129),
     2               YM(5,129),XPC(5),XMC(5),YPC(5),YMC(5),APBI(5),
     3               AMBI(5),P(5,2,33,33)
      COMMON /BLOCK4/JV(5,16),JW(5,16),SV(5,16),SW(5,16),V(5,2001,2),
     1               W(5,129,2),VS(129,2),WS(129,2)
      COMMON /BLOCK5/BUFF(2201),UI(2001),U(2001),UR(2001)
      EQUIVALENCE (BUFF(130),UT(1))
      REAL*8  S1,S2,S3,S4,S5,S6
      DO 10  I = 1, 2200
      BUFF(I) = 0.
   10 CONTINUE
C..........COMPUTE TRANSMITTED FIELD
      KM = 2 ** (NL - 1)
      CP = (C(1) + 1.) / 2.
      CM = (C(1) - 1.) / 2.
      DO 20  K = 1, KM
      JJ(K) = JV(1,K) / 2.
      JJ(KM + K) = - JW(1,K) / 2.
      S(K) = SV(1,K) * CP
      S(KM + K) = - SW(1,K) * CM
   20 CONTINUE
      DO 30  M = 2, NTGP
      WW(M) = (WS(M,1) + WS(M,2)) * H
   30 CONTINUE
      WWC = 2. * H * WS(NTGP,2)
      DIV = S(1) + H * WS(1,2)
      UT(1) = 0.
      UT(2) = UI(2) / DIV
      IM = 2 ** NL
      DO 60  K = 2, NTGP
      KP = K + 1
      S1 = 0.D0
      S2 = 0.D0
      DO 40  I = 2, K
      S1 = S1 + WW(I) * UT(2 + K - I)
   40 CONTINUE
      DO 50  I = 2, IM
      S2 = S2 + S(I) * UT(KP + JJ(I))
   50 CONTINUE
      UT(KP) = (UI(KP) - S1 - S2) / DIV
   60 CONTINUE
      KF = NTGP + 1
      S2 = 0.D0
      ITM1 = ITM + 1
      DO 90  K = KF, ITM1
      KP = K + 1
      S1 = 0.D0
      DO 70  I = 2, NTGP
```

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      S1 = S1 + WW(I) * UT(2 + K - I)
   70 CONTINUE
      S2 = S2 + UT(KP - NTGP)
      S3 = 0.D0
      DO 80  I = 2, IM
      S3 = S3 + S(I) * UT(KP + JJ(I))
   80 CONTINUE
      UT(KP) = (UI(KP) - S1 - WWC * S2 - S3) / DIV
   90 CONTINUE
C..........COMPUTE REFLECTED FIELD
      DO 91  K = 1, KM
      S(K) = - CM * S(K) / CP
      S(KM + K) = - CP * S(KM + K) / CM
   91 CONTINUE
      DO 92  M = 2, NTGP
      WW(M) = (VS(M,1) + VS(M,2)) * H
   92 CONTINUE
      WWC = 2. * H * VS(NTGP,2)
      S(1) = S(1) + H * VS(1,2)
      UR(1) = 0.
      UR(2) = S(1) * UT(2)
      DO 95  K = 2, NTGP
      KP = K + 1
      S1 = 0.D0
      S2 = 0.D0
      DO 93  I = 2, K
      S1 = S1 + WW(I) * UT(2 + K - I)
   93 CONTINUE
      DO 94  I = 1, IM
      S2 = S2 + S(I) * UT(KP + JJ(I))
   94 CONTINUE
      UR(KP) = S1 + S2
   95 CONTINUE
      S2 = 0.D0
      DO 98  K = KF, ITM
      KP = K + 1
      S1 = 0.D0
      DO 96  I = 2, NTGP
      S1 = S1 + WW(I) * UT(2 + K - I)
   96 CONTINUE
      S2 = S2 + UT(KP - NTGP)
      S3 = 0.D0
      DO 97  I = 1, IM
      S3 = S3 + S(I) * UT(KP + JJ(I))
   97 CONTINUE
      UR(KP) = S1 + S2 * WWC + S3
   98 CONTINUE
      DO 100  K = 1, ITM1
      V(NL,K,1) = UT(K)
      V(NL,K,2) = 0.
  100 CONTINUE
      CALL PRNT(3)
C..........COMPUTE FIELD AT INTERFACES
      IF (NL.EQ.1)  GO TO 160
      NTD = NL - 1
      DO 150  IL = 1, NTD
      NCL = NL - IL + 1
      NPL = NCL + 1
      NNL = NCL - 1
      CV1 =    AMBI(NCL) * (C(NPL) + 1.) / 2.
```

```
      CW1 = - APBI(NCL) * (C(NPL) - 1.) / 2.
      CV2 = - AMBI(NCL) * (C(NPL) - 1.) / 2.
      CW2 =   APBI(NCL) * (C(NPL) + 1.) / 2.
      CV3 = H * AMBI(NCL) / 2.
      CW3 = H * APBI(NCL) / 2.
      CV4 = H * XMC(NCL) * AMBI(NCL) / 4.
      CW4 = H * XPC(NCL) * APBI(NCL) / 4.
      CV5 = H * YMC(NCL) * AMBI(NCL) / 4.
      CW5 = H * YPC(NCL) * APBI(NCL) / 4.
      NP1 = NPTL(NCL)
      NPL = NPL - 1
      V(NNL,1,1) = 0.
      V(NNL,1,2) = 0.
      DO 120  K = 2, NP1
      S1 = - XM(NCL,1) * V(NPL,K,1) / 2.
      S2 = - YM(NCL,1) * V(NPL,K,2) / 2.
      S3 = - XP(NCL,1) * V(NPL,K,1) / 2.
      S4 = - YP(NCL,1) * V(NPL,K,2) / 2.
      KM1 = K - 1
      DO 110  J = 1, KM1
      KP = K + 1 - J
      S1 = S1 + XM(NCL,J) * V(NPL,KP,1)
      S2 = S2 + YM(NCL,J) * V(NPL,KP,2)
      S3 = S3 + XP(NCL,J) * V(NPL,KP,1)
      S4 = S4 + YP(NCL,J) * V(NPL,KP,2)
  110 CONTINUE
      V(NNL,K,1) = CV1 * V(NPL,K,1) + CV2 * V(NPL,K,2) + CV3 *
     1             (S1 + S2)
      V(NNL,K,2) = CW3 * (S3 + S4)
  120 CONTINUE
      S5 = 0.D0
      S6 = 0.D0
      NP2 = NP1 + 1
      NM1 = NP1 - 1
      DO 140  K = NP2, ITM1
      KM = K - NM1
      S1 = (XM(NCL,1) * V(NPL,K,1) + XM(NCL,NP1) * V(NPL,KM,1)) / 2.
      S2 = (YM(NCL,1) * V(NPL,K,2) + YM(NCL,NP1) * V(NPL,KM,2)) / 2.
      S3 = (XP(NCL,1) * V(NPL,K,1) + XP(NCL,NP1) * V(NPL,KM,1)) / 2.
      S4 = (YP(NCL,1) * V(NPL,K,2) + YP(NCL,NP1) * V(NPL,KM,2)) / 2.
      DO 130  J = 2, NM1
      KP = K + 1 - J
      S1 = S1 + XM(NCL,J) * V(NPL,KP,1)
      S2 = S2 + YM(NCL,J) * V(NPL,KP,2)
      S3 = S3 + XP(NCL,J) * V(NPL,KP,1)
      S4 = S4 + YP(NCL,J) * V(NPL,KP,2)
  130 CONTINUE
      S5 = S5 + V(NPL,KM,1) + V(NPL,K-NP1,1)
      S6 = S6 + V(NPL,KM,2) + V(NPL,K-NP1,2)
      V(NNL,K,1) = CV1 * V(NPL,K,1) + CV2 * V(NPL,K,2) +
     1             CV3 * (S1 + S2) + CV4 * S5 + CV5 * S6
      V(NNL,K,2) = CW1 * V(NPL,KM,1) + CW2 * V(NPL,KM,2) +
     1             CW3 * (S3 + S4) + CW4 * S5 + CW5 * S6
  140 CONTINUE
  150 CONTINUE
  160 CONTINUE
      H2 = H * 2.
      DO 170  I = 1, ITM1
      UT(I) = 1.
  170 CONTINUE
```

```
      IF (IOPT.EQ.1)  GO TO 300
C..........CALCULATE FIELD INSIDE MEDIUM AT TIMES T = IT * H * 2
      DO 220  IT = IFRQ, ITM, IFRQ
      IN = NTGP
      IJ = IT + 1 - (NTGP - 1) / 2
      U(IN) = V(NL,IJ,1) * UT(IJ)
      DO 210  K1 = 1, NL
      NCL = NL - K1 + 1
      NPL = NCL + 1
      NP1 = NPTL(NCL)
      NP2 = NP1 + 1
      NM1 = NP1 - 1
      CM = C(NPL) - 1.
      CP = C(NPL) + 1.
      NIT = (NP1 - 1) / 2
      DO 200  JC = 1, NIT
      IN = IN - 2
      M = NIT - JC
      M2 = M * 2
      IJ = IT - M + 1 - X(NCL) / 2
      IJM = IJ - NM1 + M2
      S1 = (V(NCL,IJ,1) * P(NCL,1,M2 + 1,1) * UT(IJ) +
     1      V(NCL,IJM,1) * P(NCL,1,NP1,NP1 - M2) * UT(IJM)) / 2.
      S2 = (V(NCL,IJM,2) * P(NCL,2,M2 + 1,1) * UT(IJM) +
     1      V(NCL,IJ,2) * P(NCL,2,NP1,NP1 - M2) * UT(IJ)) / 2.
      IF (IJ.LE.1)  GO TO 190
      JM = 2 * JC
      IF (IJM.LE.0)  JM = IJ - 1
      DO 180  J = 2, JM
      M2J = M2 + J
      S1 = S1 + V(NCL,IJ + 1 - J,1) * P(NCL,1,M2J,J)
      S2 = S2 + V(NCL,IJ + 1 - J,2) * P(NCL,2,NP2 - J,NP2 - M2J)
  180 CONTINUE
  190 CONTINUE
      MI2 = M2 + 1
      U(IN) = EAMB(NCL,MI2) * (CP * V(NCL,IJ,1) * UT(IJ) -
     1           CM * V(NCL,IJ,2) * UT(IJ) + H2 * S1)
      U(IN) = U(IN) + EAPB(NCL,MI2) * (CP * V(NCL,IJM,2) * UT(IJM) -
     1           CM * V(NCL,IJM,1) * UT(IJM) + H2 * S2)
  200 CONTINUE
  210 CONTINUE
      CALL PRNT(4)
  220 CONTINUE
      RETURN
  300 CONTINUE
C..........CALCULATE FIELD INSIDE MEDIUM AT POSITION IX
      READ (5,1000)  NOBPTS
      DO 390  KK = 1, NOBPTS
      READ (5,1000)  IX
      IF (IX - NTGP + 1)  330, 310, 330
  310 CONTINUE
      JMAX = 1 + (ITM - 1 - (IX / 2) ) / IFRQ
      DO 320  J = 1, JMAX
      U(J) = V(NL,IFRQ * (J - 1) + 1, 1)
  320 CONTINUE
      GO TO 380
  330 CONTINUE
      NLP = NL + 1
      DO 340  I = 2, NLP
      IF (IX.LE.X(I))  GO TO 350
```

```
  340 CONTINUE
      WRITE (6,2000)  IX
      RETURN
  350 CONTINUE
      NCL = I - 1
      NPL = NCL + 1
      NP1 = NPTL(NCL)
      NP2 = NP1 + 1
      NM1 = NP1 - 1
      CM = C(NPL) - 1.
      CP = C(NPL) + 1.
      NIT = (NP1 - 1) / 2
      M = (IX - X(NCL)) / 2
      M2 = M * 2
      MI2 = M2 + 1
      JC = NIT - M
      U(1) = 0.
      IMAX = 1 + (ITM - (IX / 2) ) / IFRQ
      DO 370  I = 2, IMAX
      IJ = IFRQ * (I - 1) + 1
      IJM = IJ - NM1 + M2
      S1 = (V(NCL,IJ,1) * P(NCL,1,M2 + 1,1) * UT(IJ) +
     1      V(NCL,IJM,1) * P(NCL,1,NP1,NP1 - M2) * UT(IJM)) / 2.
      S2 = (V(NCL,IJM,2) * P(NCL,2,M2 + 1,1) * UT(IJM) +
     1      V(NCL,IJ,2) * P(NCL,2,NP1,NP1 - M2) * UT(IJ)) / 2.
      JM = 2. * JC
      IF (IJM.LE.0)  JM = IJ - 1
      DO 360  J = 2, JM
      M2J = M2 + J
      S1 = S1 + V(NCL,IJ + 1 - J,1) * P(NCL,1,M2J,J)
      S2 = S2 + V(NCL,IJ + 1 - J,2) * P(NCL,2,NP2 - J,NP2 - M2J)
  360 CONTINUE
      MI2 = M2 + 1
      U(I) = EAMB(NCL,MI2) * (CP * V(NCL,IJ,1) * UT(IJ) -
     1         CM * V(NCL,IJ,2) * UT(IJ) + M2 * S1)
      U(I) = U(I) + EAPB(NCL,MI2) * (CP * V(NCL,IJM,2) * UT(IJM) -
     1         CM * V(NCL,IJM,1) * UT(IJM) + M2 * S2)
  370 CONTINUE
  380 CONTINUE
      IT = IX
      CALL PRNT(5)
  390 CONTINUE
      RETURN
 1000 FORMAT (I5)
 2000 FORMAT (///,1X,9HHEY, IX =,I5,14H  IS TOO LARGE)
      END
```

VIII. SETTING UP SOME PROGRAMS

Consider a three-layer medium of depth 3 cm, with each layer being 1 cm deep. Assume that the permittivity and conductivity are constant on each layer and are given by the values shown in Figure B-3.

N is chosen to be 128. This is not for purposes of accuracy (N=64 yields identical results) but rather for obtaining more grid points, resulting in smoother plots.

Turning to the computation of the X(i)'s, note first that

$$\ell=(0.01)(\sqrt{30}+\sqrt{135}+\sqrt{60})/c$$

where c is the speed of light in free space. Then X(1)=0 and

$$x_1=\sqrt{30}/(\sqrt{30}=\sqrt{135}+\sqrt{60})=X(2)/128.$$

Thus, X(2)=28.22 ··· and so, upon rounding

$$X(2)=28.$$

This is equivalent to using

$$\varepsilon_1(z)=29.4\varepsilon_0 \text{ rather then } \varepsilon_1(z)=30\varepsilon_0.$$

Now

$$x_2=(\sqrt{29.4}+\sqrt{135})/(\sqrt{29.4}+\sqrt{135}+\sqrt{60})$$
$$=X(3)/128$$

and so

$$X(3)=88$$

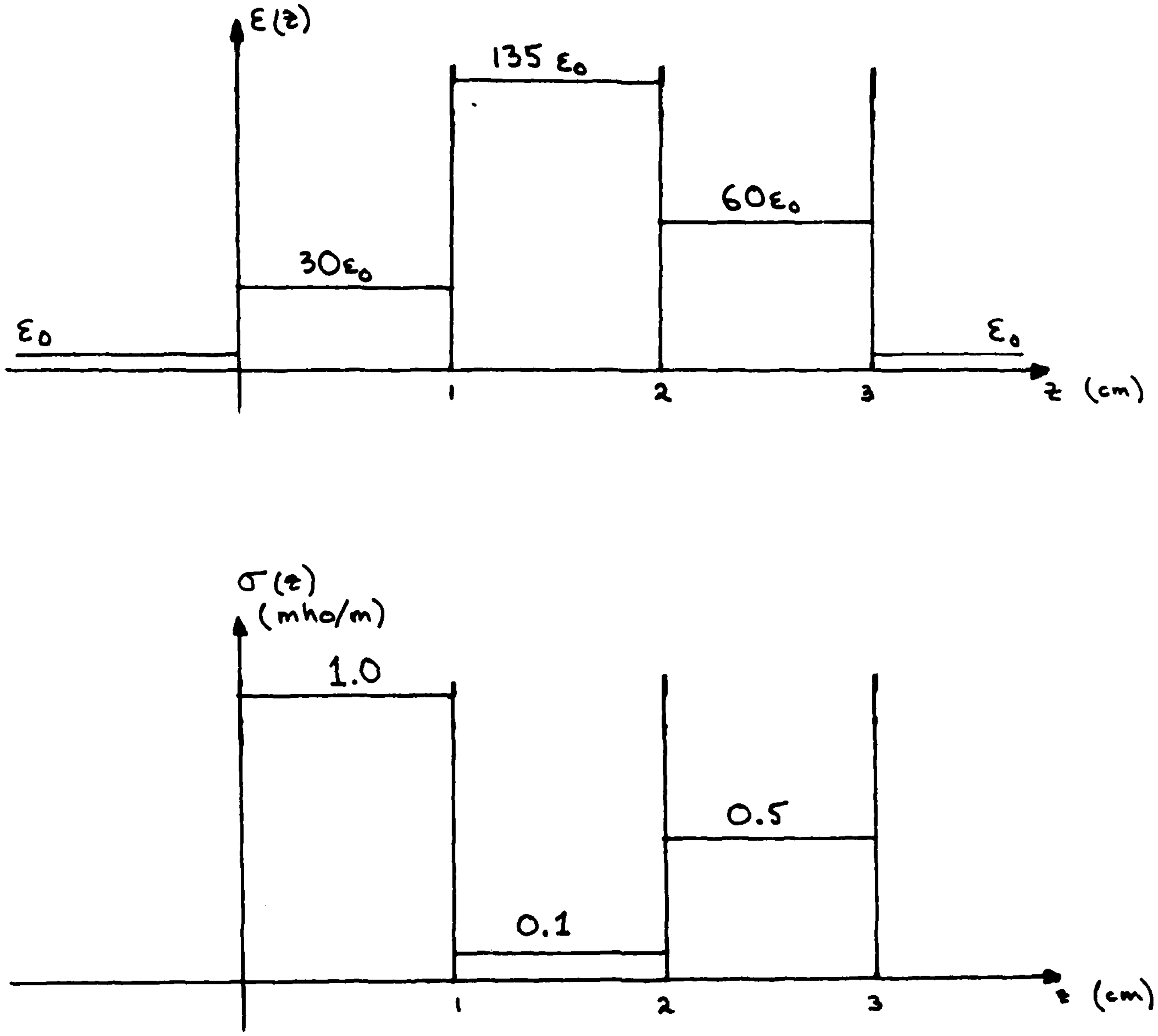

Figure B-3. Permittivity and conductivity profiles for a three-layer medium.

and

$$X(4)=128.$$

Since

$$X(4)-X(3)>32$$

$$X(3)-X(2)>32$$

two additional layers need to be inserted into the medium. (Note that if N=64, no additional layers are needed.) Hence, set

$$X(1)=0$$

$$X(2)=28$$

$$X(3)=60$$

$$X(4)=88$$

$$X(5)=108$$

$$X(6)=128.$$

Now

$$C(1)=\sqrt{29.4}$$

$$C(2)=\sqrt{135/29.4}$$

$$C(3)=1.0$$

$$C(4)=\sqrt{60/135}$$

$$C(5)=1.0$$

$$C(6)=\sqrt{1/60}.$$

The only nonzero functions to be entered into subroutine EVAL are

$$B1(x)=-(1.0)\ell/(29.4\varepsilon_0)$$

$$B2(x)=B3(x)=-(0.1)\ell/(135\varepsilon_0)$$

$$B4(x)=B5(x)=-(0.5)\ell/(60\varepsilon_0)$$

A 1000-MHz sinusoidal incident field of duration 20 ns is used. Thus,

$$E^{i}(-t)=\sin(\omega t)\cdot H_{e}(20\cdot 10^{-9}-t)$$

where $\omega=2\pi\cdot 10^{9}$ and H_e is the Heaviside function. Since $\tau=t/\ell$, it follows that the incident field to be used in INCWAV is

$$\mathrm{UINC}(\tau)=\sin(-\omega\ell\tau).$$

Now the τ step size is

$$\Delta\tau=2H=1/64$$

so it follows that the t step size is

$$\Delta t=\ell/64=.01291 \text{ ns}.$$

The period of the incident signal is 1 ns which corresponds to approximately 77.5 Δt. In light of this the incident signal was modified so that its period was an even multiple of Δt. (This modification is not at all necessary and was performed solely to make numbers turn out "nice.") The period of E^{i} was lengthened to 78 Δt which corresponds to a 993-MHz signal. Then UINC can be written

$$\mathrm{UINC}(\tau)=\sin(-\pi\cdot\tau/(78\cdot H)).$$

(See Section IX for a source listing of INCWAV for this particular field.)

The duration of this new incident field was lengthened to 20.13951 ns so that 20 cycles would still be considered. Thus, ITM should be at least 1560 so that signal cutoff is observed. It was decided to print out the entire interior field every half period, so IFRQ was set equal to 39 and ITM was chosen to be 1950. Setting ISCAT=1 and IOPT=0, the scattered and internal fields were put onto disk storage. These fields were then graphed using the routine given in Section X.

The above results indicate that steady state was essentially attained after six periods of the incident field; i.e., after 468 $\Delta\tau$ intervals. It was then decided to search for the maximum intensity occurring at each grid point during the passing of the transient field (first 6 cycles) and during steady state. Thus, HATS was run a second time, with IFRQ=1, ITM=468, ISCAT=0, and IOPT=0 and with the modified subroutine PRNT given in Section XI. To determine the maximum steady state intensities, HATS was run with IFRQ=780 and ITM=858. Upon the first call to PRNT using PRNT(4), the value of IFRQ was changed to 1 so that the entire period was examined in as much detail as possible. Results from these runs were plotted on the same set of axes.

From these plots it was observed that two points in the medium (one in the first layer, one in the third) had a considerably stronger transient field than steady field. Thus, a fourth run of HATS was made looking at the detailed time behavior at these points as well as at a point in the second layer which seemed to have almost no difference in transient vs. steady state intensity. Thus, three more plots were generated with the parameters used in HATS being IFRQ=3; ITM=1950; ISCAT=0; IOPT=1; NOBPTS=3; and IX=20, 32, 92.

For the sake of interest this entire procedure was pursued a second time for a nonconducting medium. This was achieved by setting B1(x)=B2(z)=... =B5(x)=0.

Finally, so that the internal reflections in the medium in the nonconducting case could be observed, a triangular incident spike field was used:

UI(1)=0

UI(2)=0.5

UI(3)=1.0

UI(4)=0.5

UI(5)=UI(6)=••••=0.

Plots of all output appear in Section XII.

IX. SOURCE LISTING OF INCWAV FOR FIELD OF FINITE DURATION

```
      SUBROUTINE INCWAV
C..........INSERT FUNCTION STATEMENT FOR INCIDENT FIELD
      UINC(X) = SIN( - PI * X / (78. * H) )
C..........
      COMMON /BLOCK1/H,NL,NTGP,NPTL(5),IFRQ,ITM,IT,ISCAT,IOPT
      COMMON /BLOCK5/BUFF(2201),UI(2001),U(2001),UR(2001)
      PI = 3.1415927
      H2 = H * 2.
      ITM1 = ITM + 1
      DO 10  J = 1, ITM1
      UI(J) = 0.
   10 CONTINUE
      DO 20  J = 2, 39
      X = - (J - 1) * H2
      UI(J) = UINC(X)
      UI(39 + J) = - UI(J)
   20 CONTINUE
      DO 40  J = 2, 20
      JC = (J - 1) * 78
      DO 30  I = 2, 78
      UI(JC + I) = UI(I)
   30 CONTINUE
   40 CONTINUE
      RETURN
      END
```

X. SOURCE LISTING OF A PLOTTING ROUTINE

```
      DIMENSION X(193),A1(2100),A2(65),A3(2200),U(193)
      DIMENSION X0(2),X1(2),X2(2),X3(2),X4(2),U0(2),U1(2),U2(2),
     1          U3(2),U4(2)
      READ (8,2000)  N, NL, IFRQ, ITM
      N = N / 2
      H2 = 1. / N
      NP1 = N + 1
      N2 = N * 2
      N21 = N2 + 1
      MN = N * 3 + 1
      DO 10  I = 1, MN
      X(I) = -1. + (I - 1) * H2
   10 CONTINUE
      DO 20  I = 1, N
      A1(I) = 0.
   20 CONTINUE
      DO 30  I = 1, N2
      A3(I) = 0.
   30 CONTINUE
      ITM1 = ITM + 1
      L = ITM1 + N2
      READ (8,1500)  (A3(I), I = N21, L)
      L = ITM1 + N
      READ (8,1500)  (A1(I), I = NP1, L)
C..........ESTABLISH ARRAYS TO BE USED IN GRAPHING BOUNDARIES OF
C..............LAYERS AND HORIZONTAL AXIS
```

```
      X0(1) = 0.
      X0(2) = X0(1)
      X1(1) = .21875
      X1(2) = X1(1)
      X2(1) = .6875
      X2(2) = X2(1)
      X3(1) = 1.0
      X3(2) = X3(1)
      X4(1) = -1.
      X4(2) =  2.
      U0(1) = -1.2
      U0(2) =  1.2
      U1(1) = -.75
      U1(2) =  .75
      U4(1) = 0.
      U4(2) = 0.
      DO 100  IT = IFRQ, ITM, IFRQ
      READ (8,6000)  K
      READ (8,1500)  (A2(I), I = 1, NP1)
      DO 40  I = 1, NP1
      U(I) = A1(IT + I)
      U(N2 + I) = A3(NP1 + IT - I + 1)
   40 CONTINUE
      DO 50  I = 1, NP1
      U(N + I) = A2(I)
   50 CONTINUE
C..........THE ARRAY U NOW CONTAINS THE REFLECTED, INTERNAL AND
C..............TRANSMITTED FIELDS AT TIME TAU = IT * 2 * H
C..........PLOT ARRAY U
      CALL GRAPH (MN, X, U, 0, 2,  6.01, 4.01,.50,-1.,.60,-1.2,
     1          'NORMALIZED DEPTH;', 'NORMALIZED INTENSITY',
     2          ';', ';')
C..........PLOT VERTICAL LINES SHOWING POSITION OF LAYERS
      CALL GRAPHS (2, X0, U0, 0, 4, ';')
      CALL GRAPHS (2, X1, U1, 0, 4, ';')
      CALL GRAPHS (2, X2, U1, 0, 4, ';')
      CALL GRAPHS (2, X3, U0, 0, 4, ';')
C..........PLOT HORIZONTAL AXIS AT ZERO INTENSITY
      CALL GRAPHS (2, X4, U4, 0, 4, ';')
  100 CONTINUE
      STOP
 1500 FORMAT (8E10.3)
 2000 FORMAT (4I5)
 6000 FORMAT (I10)
      END
```

XI. SEARCHING FOR MAXIMUM INTENSITY

The following source listing is a modification of subroutine PRNT used to search the program output for maximum intensity.

```
      SUBROUTINE PRNT(K)
      INTEGER X
      COMMON /BLOCK1/H,NL,NTGP,NPTL(5),IFRQ,ITM,IT,ISCAT,IOPT
      COMMON /BLOCK2/A(33),B(33),AP(33),BP(33),DT(33),AH(33),BH(33),
     1              APH(33),BPH(33),DTH(33),AI(33),BI(33),DTI(33),
     2              DP(33),EBI(33),AR(5),AL(5),BR(5),BL(5)
      COMMON /BLOCK3/X(6),C(6),DPI(33),DPBI(33),EAMB(5,33),EAPB(5,33),
     1              QOLD(33),QNEW(33),XP(5,129),XM(5,129),YP(5,129),
     2              YM(5,129),XPC(5),XMC(5),YPC(5),YMC(5),APBI(5),
     3              AMBI(5),P(5,2,33,33)
      COMMON /BLOCK4/JV(5,16),JW(5,16),SV(5,16),SW(5,16),V(5,2001,2),
     1              W(5,129,2),VS(129,2),WS(129,2)
      COMMON /BLOCK5/BUFF(2201),UI(2001),U(2001),UR(2001)
      DIMENSION EMAX(129),ITIME(129)
      IF (K - 2)  1, 2, 50
    1 CONTINUE
C..........PRINT OUT INPUT DATA FROM LINES 1, 2, 3
      N = NTGP -1
      WRITE (6,2000)  N, NL, IFRQ, ITM, ISCAT, IOPT, (X(I), I = 1, NL)
      WRITE (6,3000)  (C(I),I = 1, NL)
C      WRITE (8,2000)  N, NL, IFRQ, ITM
      RETURN
    2 CONTINUE
      RETURN
   50 CONTINUE
      IF (K - 4)  3, 4, 60
    3 CONTINUE
C..........PRINT INCIDENT, REFLECTED AND TRANSMITTED FIELDS IF ISCAT = 1
      IF (ISCAT .EQ. 0)  RETURN
      ITM1 = ITM + 1
      WRITE (6,5000)
      WRITE (6,1000)  (UI(I),I = 1, ITM1)
      WRITE (6,5100)
      WRITE (6,1000)  (UR(I),I = 1, ITM1)
      WRITE (6,5200)
      WRITE (6,1000)  (V(NL,I,1),I = 1, ITM1)
C      WRITE (8,1500)  (V(NL,I,1),I = 1, ITM1)
C      WRITE (8,1500)  (UR(I),I = 1, ITM1)
      RETURN
    4 CONTINUE
C..........SEARCH FOR MAXIMUM INTENSITY AT EACH GRID POINT AND RECORD
C.............THE TIME AT WHICH IT OCCURS
      IF (IT.GT.IFRQ)  GO TO 110
      DO 100  I = 1, NTGP, 2
      EMAX(I) = 0.
  100 CONTINUE
  110 CONTINUE
      DO 120  I = 1, NTGP, 2
      ABSV = ABS( U(I) )
      IF (ABSV.LT.EMAX(I))  GO TO 120
      EMAX(I) = ABSV
      ITIME(I) = IT
  120 CONTINUE
      IF (IT.LT.ITM)  RETURN
      WRITE (6,1500)  (EMAX(I),I = 1, NTGP, 2)
      WRITE (6,5000)
      WRITE (6,9000)  (ITIME(I),I = 1, NTGP, 2)
C      WRITE (8,1500)  (EMAX(I),I = 1, NTGP, 2)
C      WRITE (8,9000)  (ITIME(I),I = 1, NTGP, 2)
      RETURN
```

```
   60 CONTINUE
      IF (K - 6)  5, 6, 6
    5 CONTINUE
C..........PRINT FIELD AT POSITION X = IT * H  FOR TIMES
C............TAU = IT * H, (IT + IFRQ * 2) * H, ...
      IMAX = 1 + (ITM - (IT / 2) ) / IFRQ
      WRITE (6,8000)  IT
      WRITE (6,1000)  (U(I),I = 1, IMAX)
C      WRITE (8,6000)  IT
C      WRITE (8,1500)  (U(I),I = 1, IMAX)
      RETURN
    6 CONTINUE
      WRITE (6,4000)
      RETURN
 1000 FORMAT (1X,8E10.3)
 1500 FORMAT (8E10.3)
 2000 FORMAT (6I5,/,6I5)
 3000 FORMAT (6E12.5)
 4000 FORMAT (///,1X,8HFINISHED)
 5000 FORMAT (///,1X,14HINCIDENT FIELD,/)
 5100 FORMAT (///,1X,15HREFLECTED FIELD,/)
 5200 FORMAT (///,1X,17HTRANSMITTED FIELD,/)
 6000 FORMAT (I10)
 7000 FORMAT (///,1X,29HINTERNAL FIELD AT TIME TAU = ,I10,8H * 2 * H,/)
 8000 FORMAT (///,1X,21HFIELD AT POSITION X =,I10,4H * H,/)
 9000 FORMAT (8I10)
      END
```

XII. PLOTS FROM SAMPLE PROGRAMS

Internal, reflected, and transmitted fields for mediums with nonzero and zero conductivity. Incident field is 1000-MHz pulse of duration 20 ns, impinging on the medium from the left at time t=0.

	Nonzero conductivity	Zero conductivity
t=6 ns	Figure A-4	Figure A-10
t=21 ns	" A-5	" A-11

Comparison of transient and steady state intensities in mediums with nonzero and zero conductivity. Incident field is 1000-MHz pulse.

Nonzero conductivity	Zero conductivity
Figure A-6	Figure A-12

Internal fields, nonzero and zero conductivity.

	Nonzero conductivity	Zero conductivity
x=.15625	Figure A-7	Figure A-13
x=.25	" A-8	" A-14
x=.71875	" A-9	" A-15

Reflected and internal fields, zero conductivity, spike incident field.

t=.181 ns	Figure A-16
t=.361 ns	" A-17
t=.542 ns	" A-18
t=.723 ns	" A-19

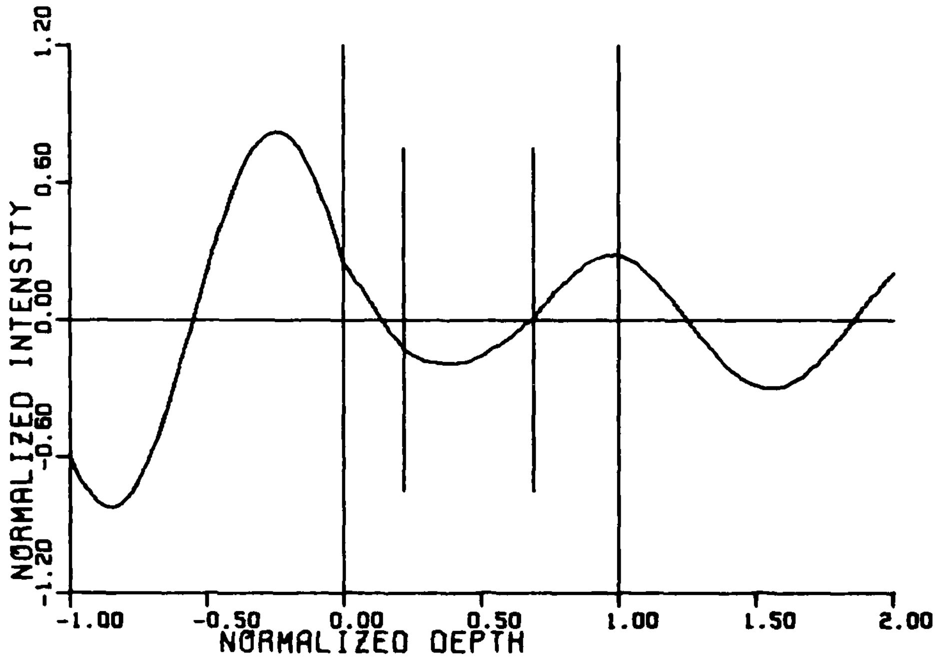

Figure B-4. Internal, reflected, and transmitted fields for a medium with nonzero conductivity, time t=6 ns. Incident field is 1000-MHz pulse of 20-ns duration, impinging on the medium from the left at time t=0.

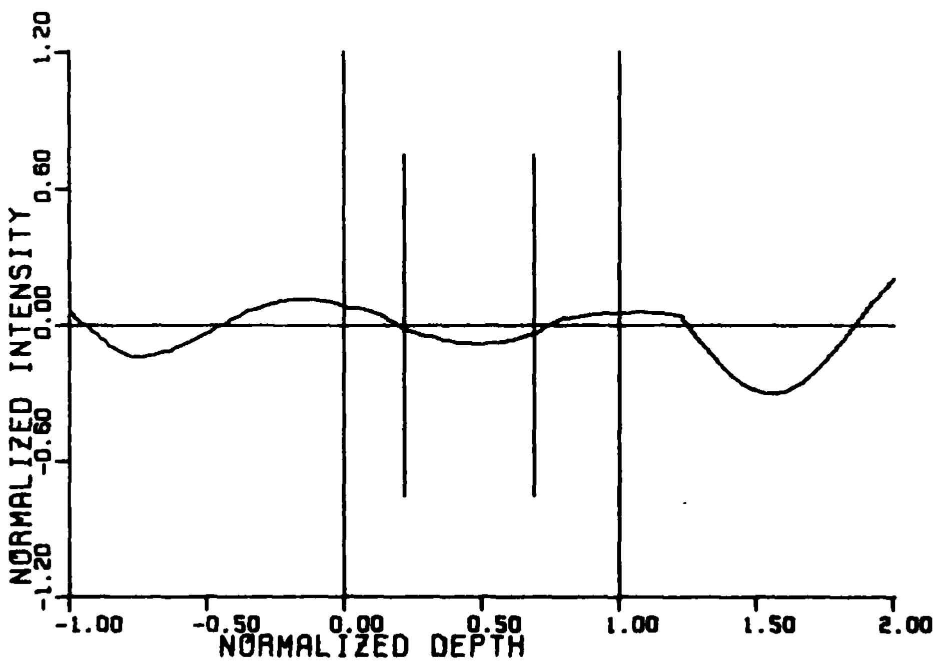

Figure B-5. Internal, reflected, and transmitted fields for a medium with nonzero conductivity, time t=21 ns. Incident field is 1000-MHz pulse of 20-ns duration, impinging on the medium from the left at time t=0.

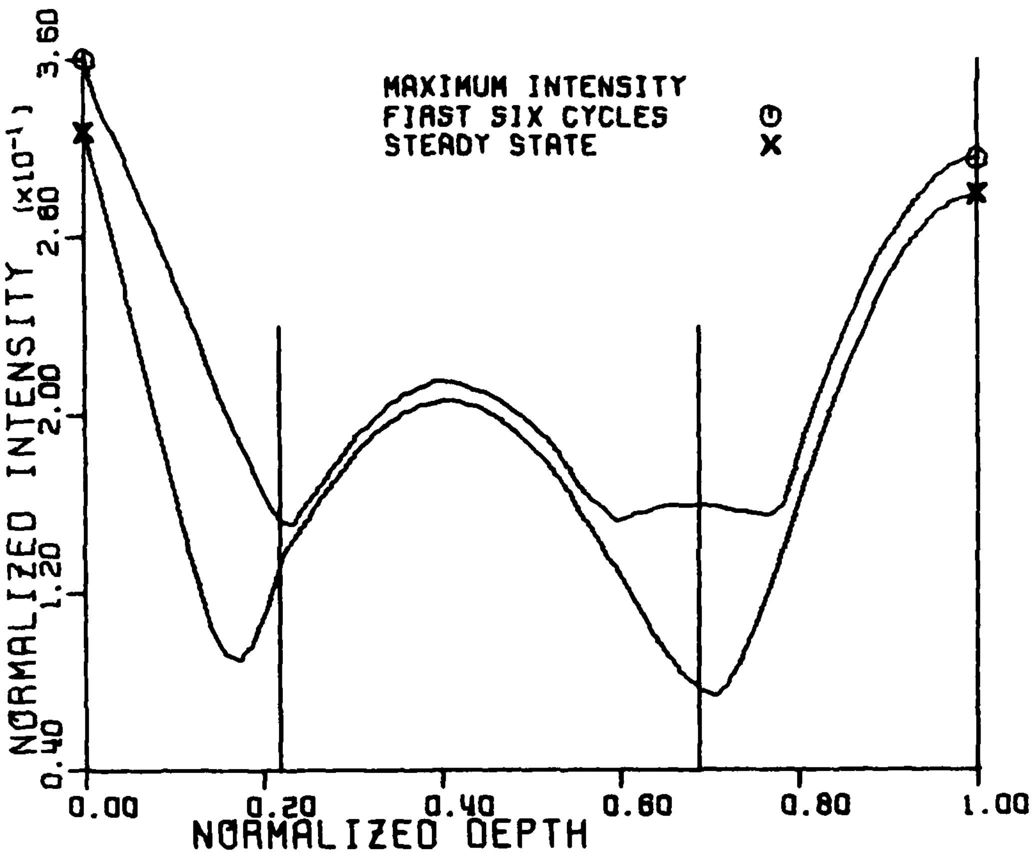

Figure B-6. Comparison of transient and steady state intensities in a medium with nonzero conductivity. Incident field is 1000-MHz pulse.

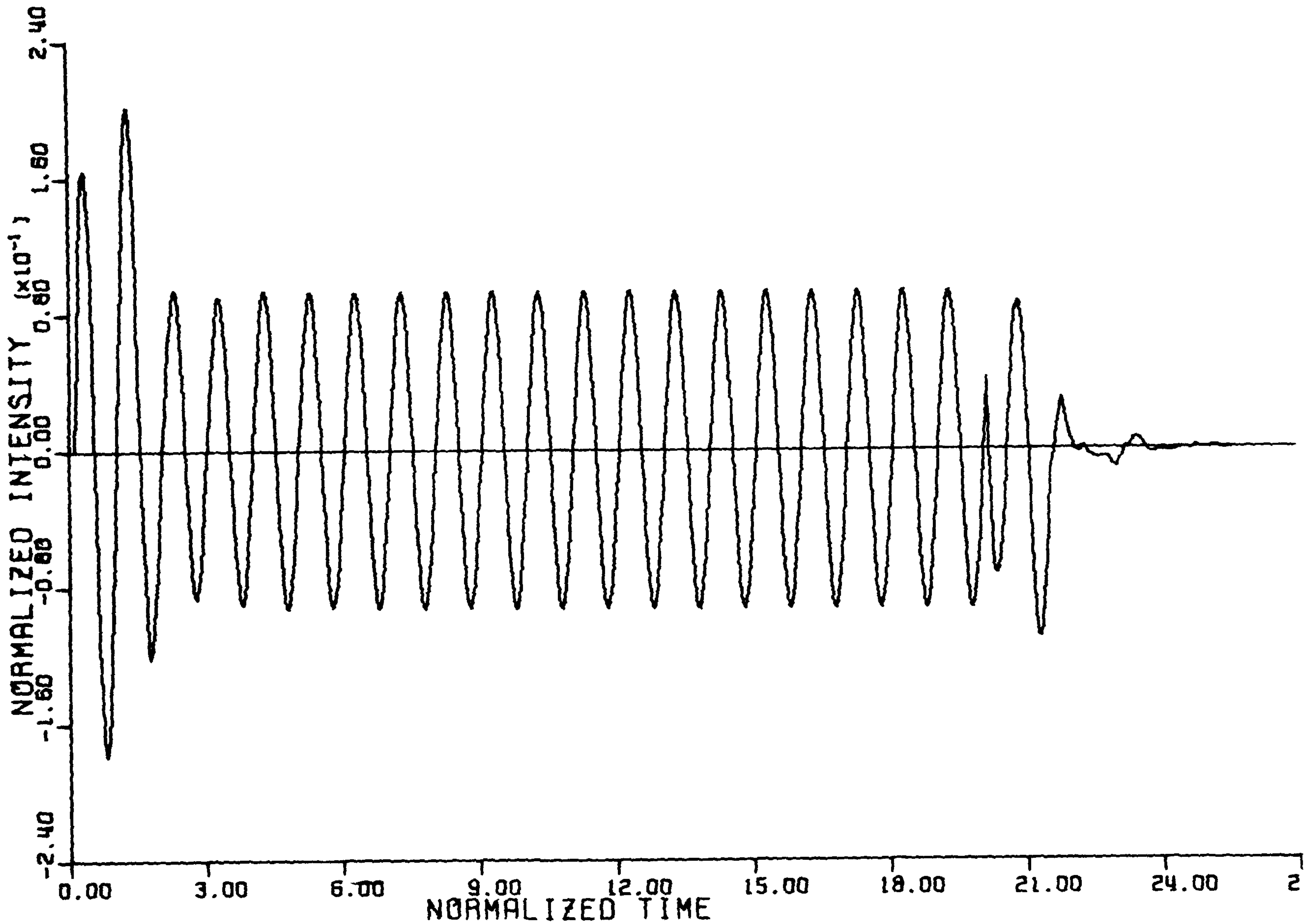

Figure B-7. Internal field (x=.15625), nonzero conductivity.

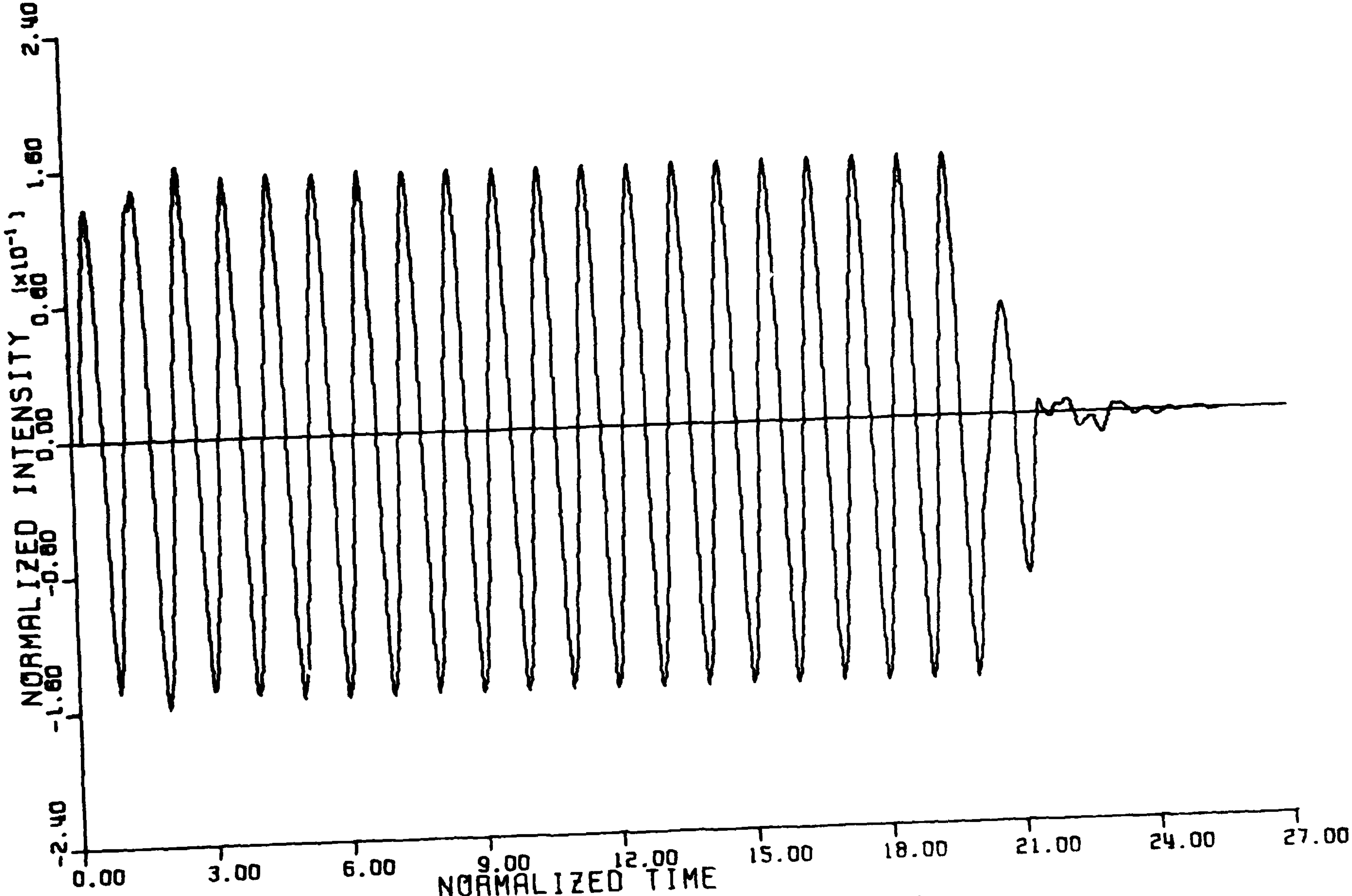

Figure B-8. Internal field (x=.25), nonzero conductivity.

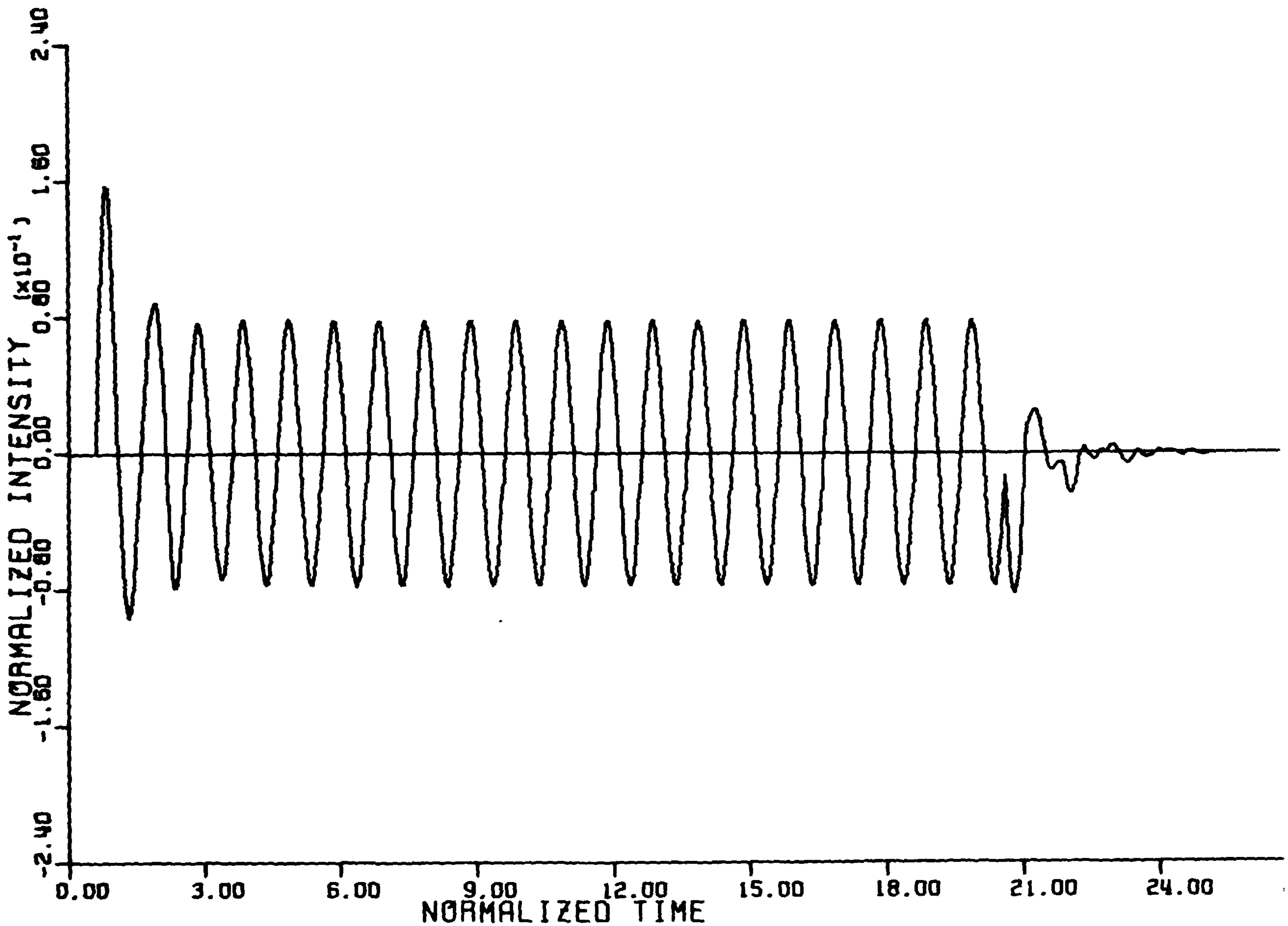

Figure B-9. Internal field (x=.71875), nonzero conductivity.

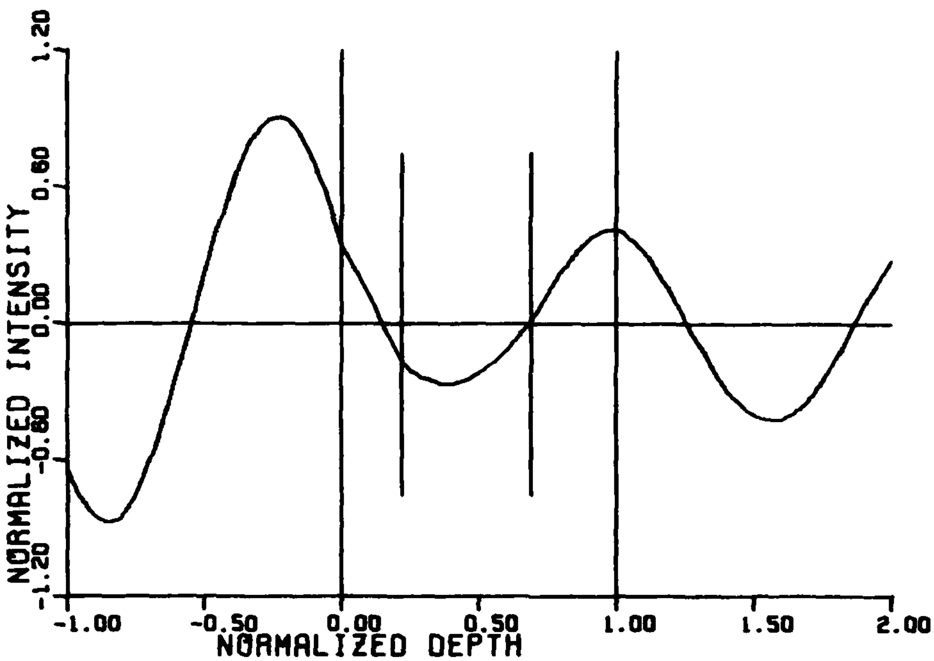

Figure B-10. Internal, reflected, and transmitted fields for a medium with zero conductivity, time t=6 ns. Incident field is 1000-MHz pulse of 20-ns duration, impinging on the medium from the left at time t=0.

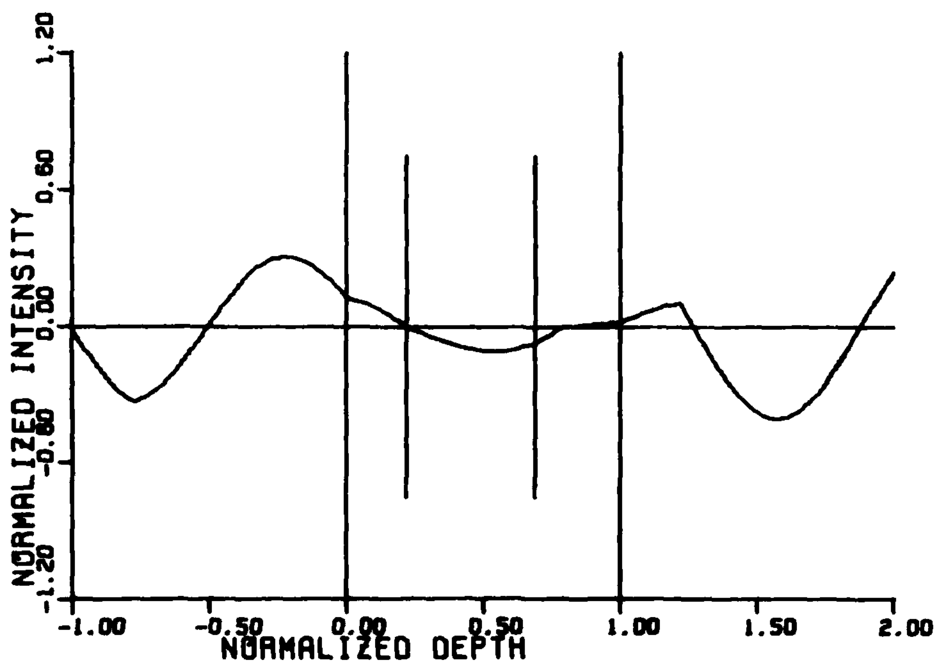

Figure B-11. Internal, reflected, and transmitted fields for a medium with zero conductivity, time t=21 ns. Incident field is 1000-MHz pulse of 20-ns duration, impinging on the medium from the left at time t=0.

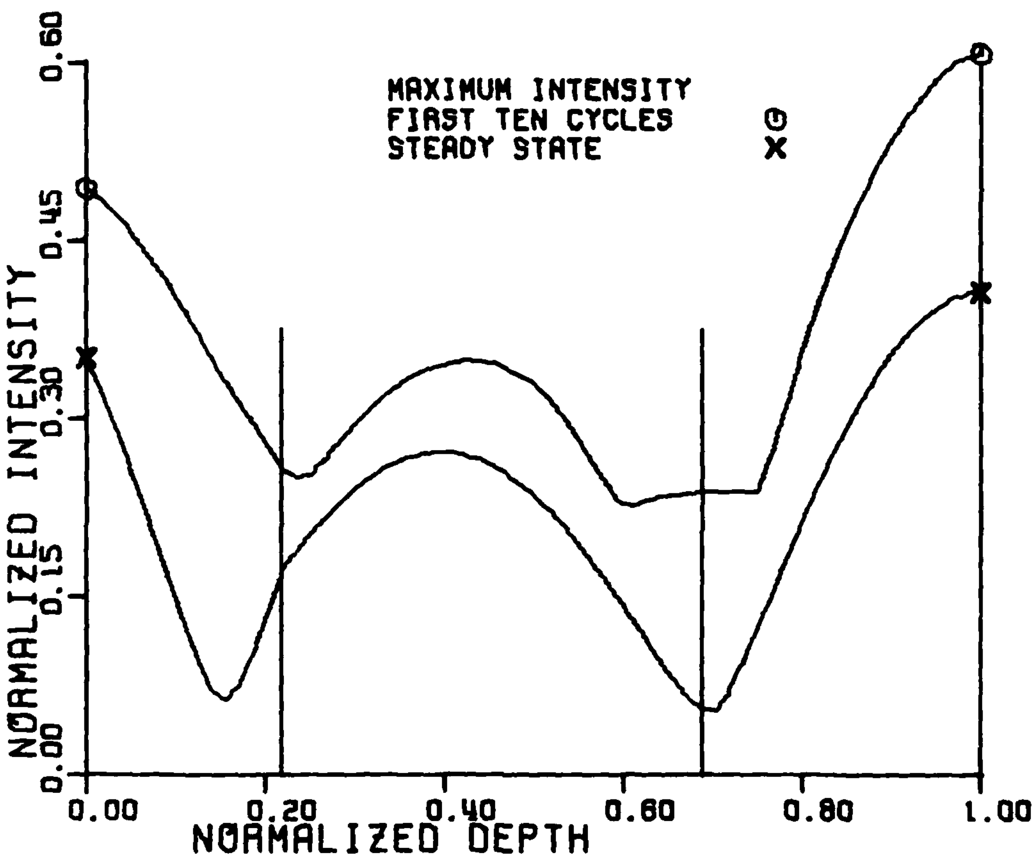

Figure B-12. Comparison of transient and steady state intensities in a medium with zero conductivity. Incident field is 1000-MHz pulse.

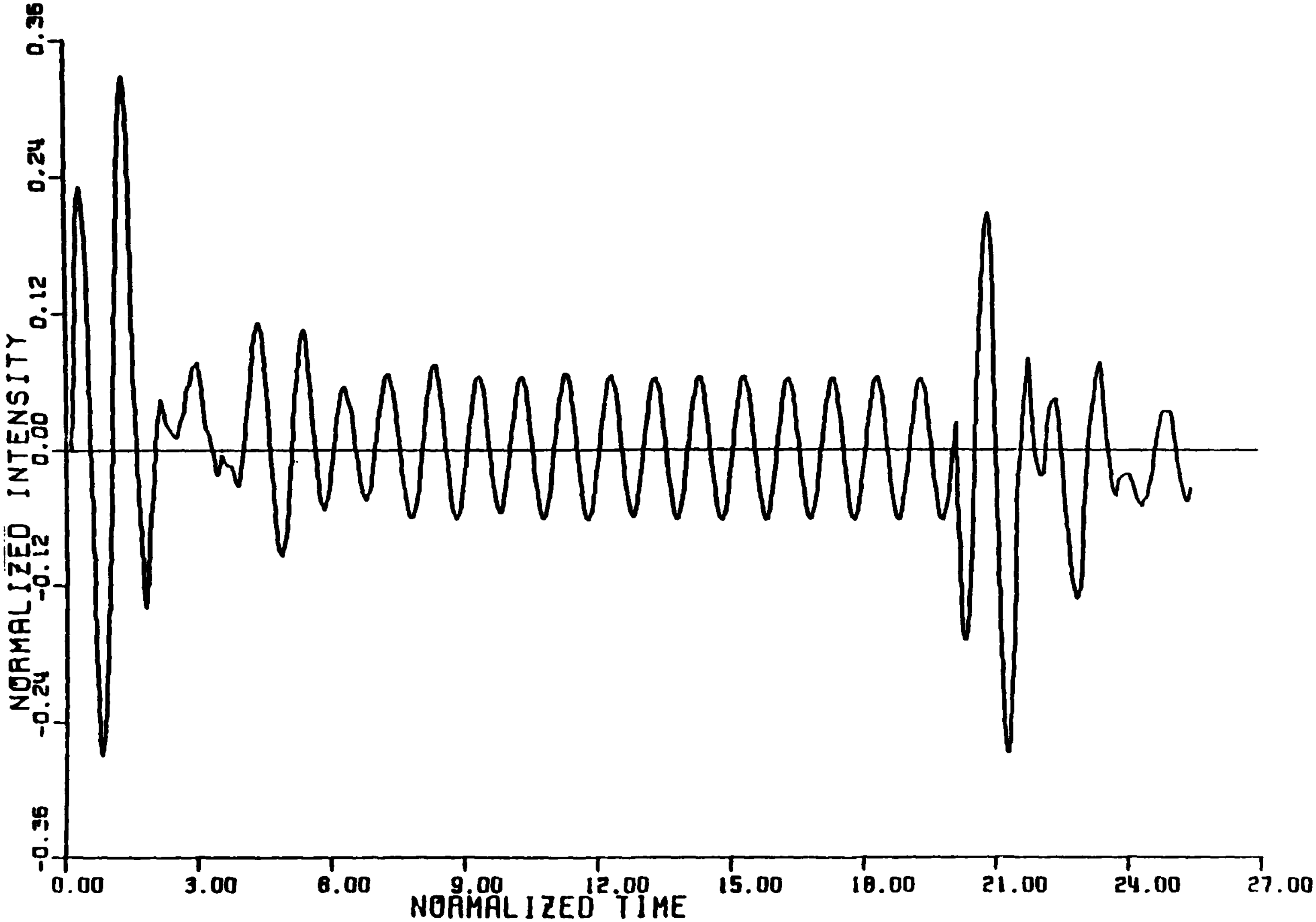

Figure B-13. Internal field (x=.15625), zero conductivity.

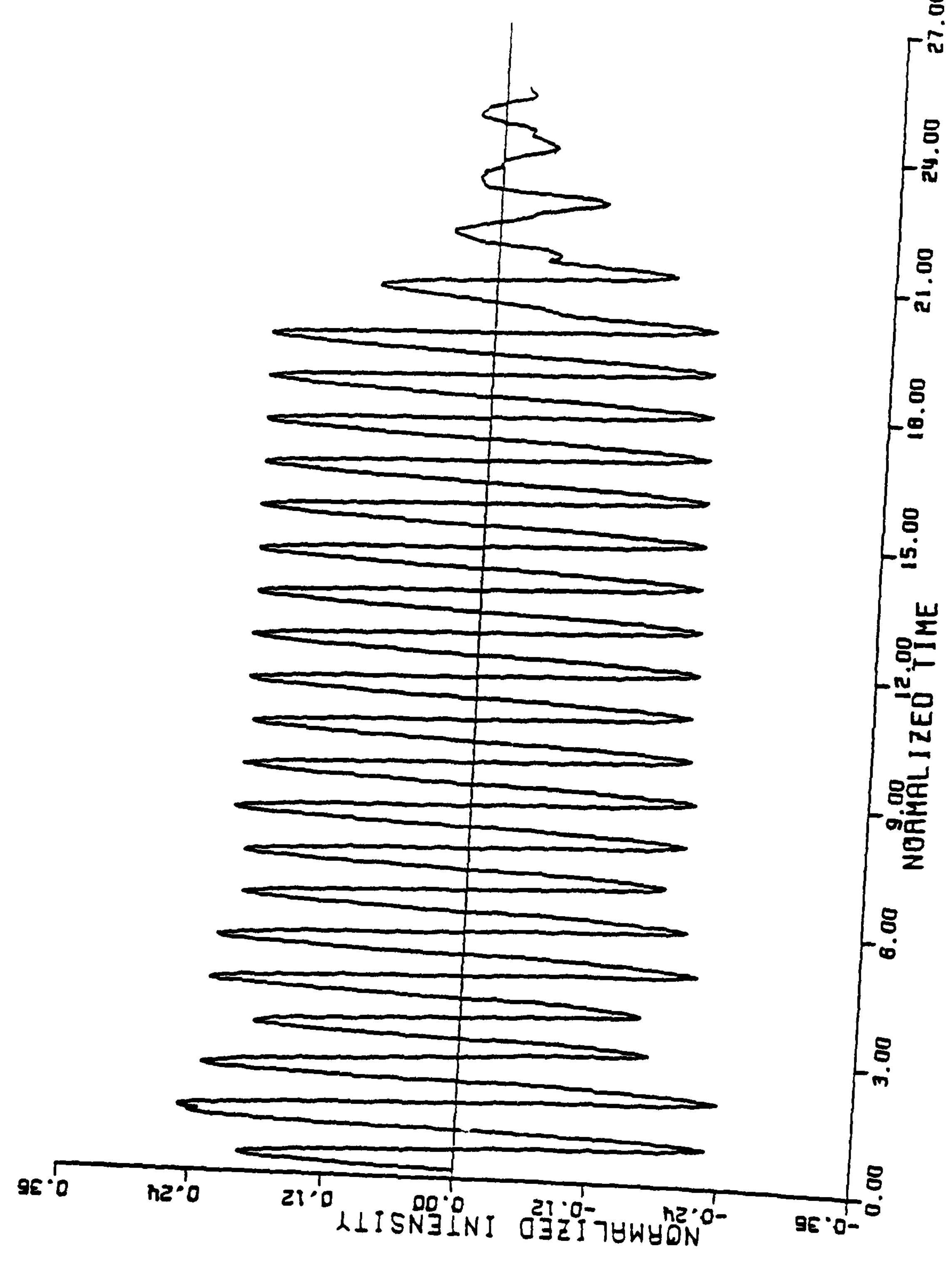

Figure B-14. Internal field (x=.25), zero conductivity.

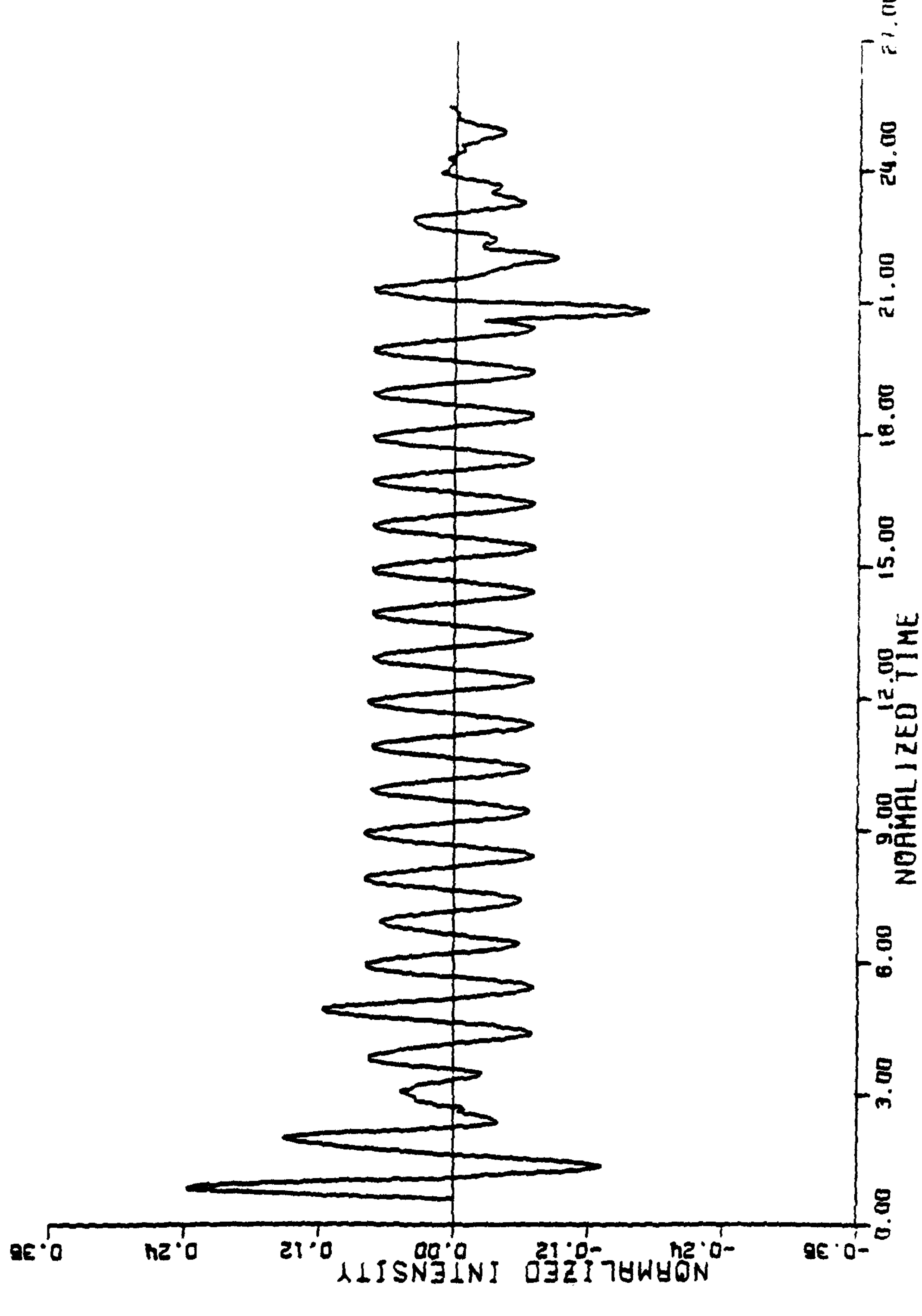

Figure B-15. Internal field (x=.71875), zero conductivity.

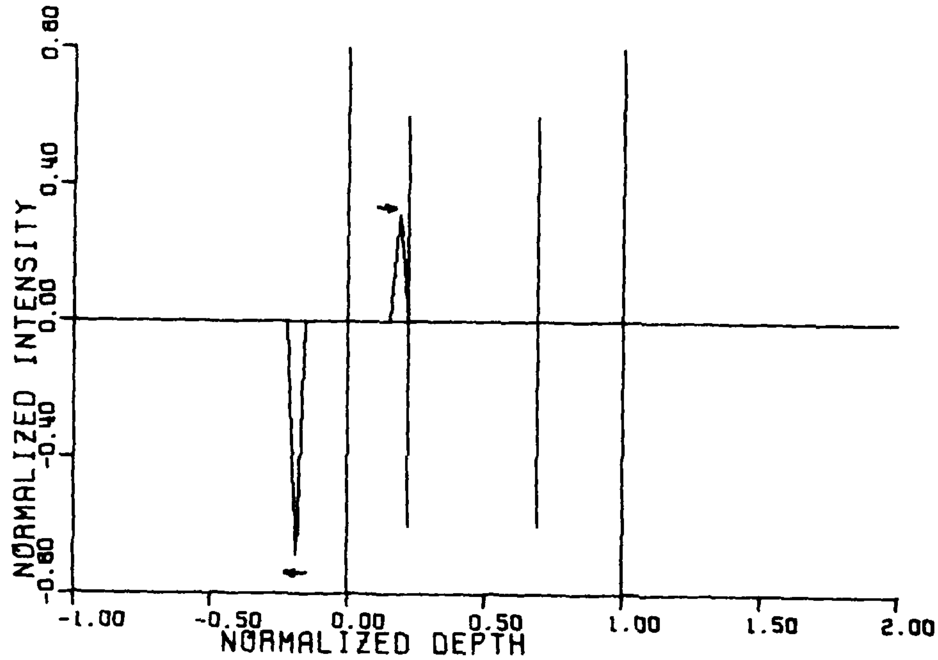

Figure B-16. Reflected and internal fields (t=.181 ns), zero conductivity, spike incident field.

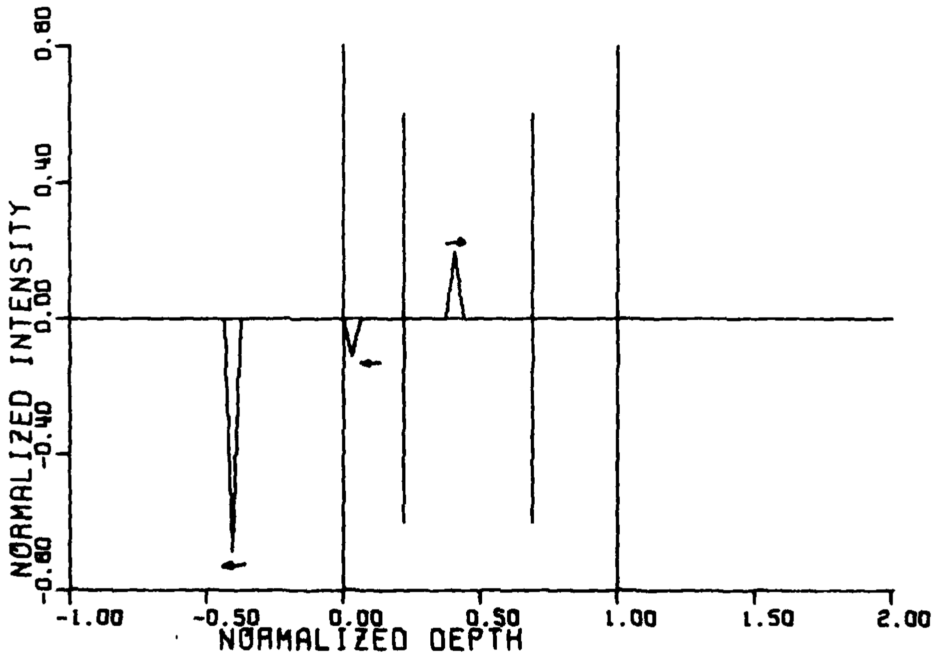

Figure B-17. Reflected and internal fields (t=.361 ns), zero conductivity, spike incident field.

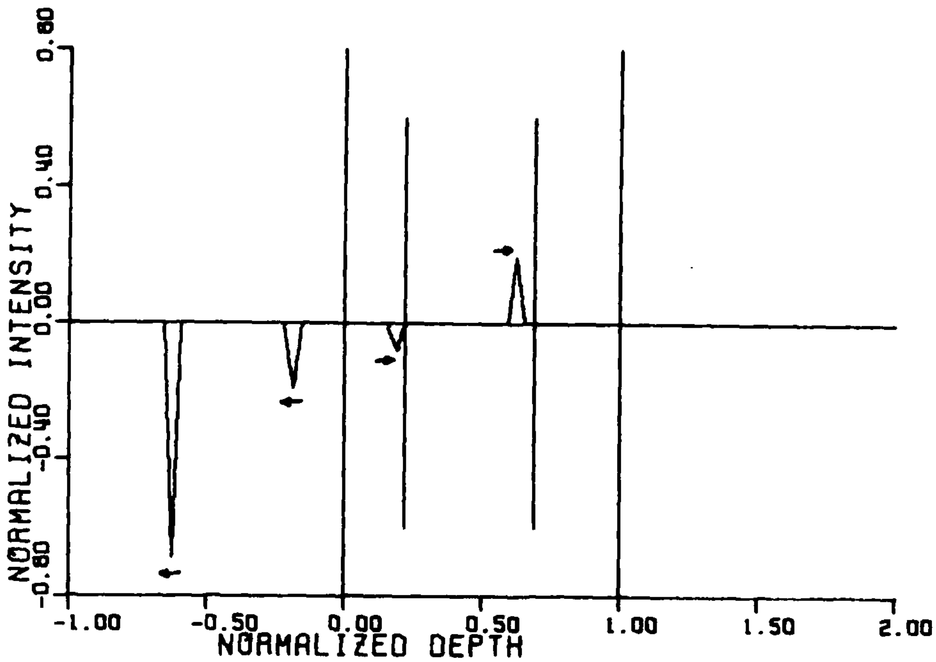

Figure B-18. Reflected and internal fields (t=.542 ns), zero conductivity, spike incident field.

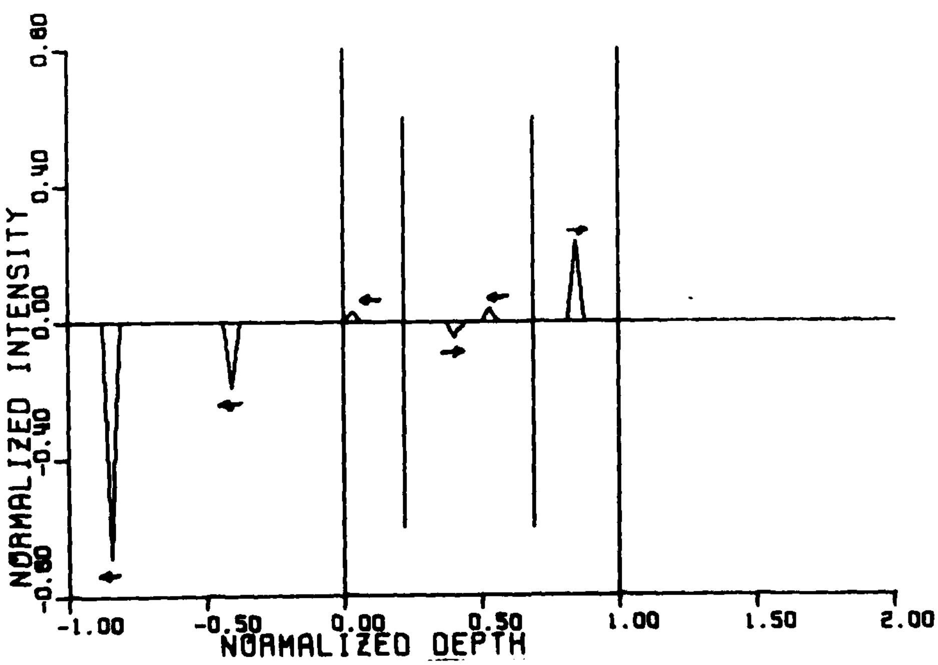

Figure B-19. Reflected and internal fields (t=.723 ns), zero conductivity, spike incident field.

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